1. Electrostatics

1. What is Electrostatics?

Electrostatics is the branch of electricity, which deals with stationary charges.

2. What is triboelectric charging?

Charging the objects through rubbing is called triboelectric charging.

3. What are the basic properties of charges?

- Electric charge.
- Conservation of charges.
- Quantisation of charges.

4. What is meant by Electric charge? Give its unit.

Electric charge is the intrinsic and fundamental property of particles. The SI unit of charge is coulomb.

5. What is Conservation of charges?

The total electric charge in the universe is constant and charge can neither be created nor be destroyed. In any physical process, the net change in charge will always be zero. This is called conservation of charges.

6. What is meant by Quantisation of charges?

- The fundamental unit of charge is 'e'. The charge q on any object is equal to an integral multiple of this fundamental unit of charge 'e'.
- *i.e.* **q = ne**, Here n is any integer (0, ±1, ±2, ±3, ±4,...). This is called quantisation of electric charge.

7. State Coulomb's law.

Coulomb's law states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between those two point charges.

8. Write down Coulomb's law in vector form and mention what each term represents.

$$\textbf{Vector form}: \vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$$

Where,

$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2C^{-2}$,proportional contstant.

q_1 , q_2 – Point charges.

r – The distance between q_1 and q_2.

$\hat{r}$ – The unit vector directed from one charge to other

9. Define 1 Coulomb.

One Coulomb is defined as the quantity of charge, which when placed at a distance of 1 metre in air or vacuum from an equal charge, experiences a electrostatic force of 9×10^9 N.

10. Write a short note on permittivity.

Permittivity is the capacity of a medium to permit electric field lines. Permittivity of free space or vaccum is constant. Its value is $\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$.

11. Define relative permittivity of a medium.

- Relative permittivity of a medium is defined as the ratio of permittivity of the medium(ε) to the permittivity of free space(ε_0) .

$$i.e. \quad \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

- For vacuum or air, $\varepsilon_r = 1$ and for all other media $\varepsilon_r > 1$.

12. What are the differences between Coulomb and gravitational forces?

S. No	Coulomb force	Gravitational force
1.	Coulomb force between two charges can be attractive or repulsive, depending on the nature of charges.	The gravitational force between two masses is always attractive.
2.	Coulomb law constant $k = 9 \times 10^9$ N m² C⁻².	The value of the gravitational constant $G = 6.626 \times 10^{-11}$ N m² kg⁻².
3.	Since k > G, Coulomb force is greater than Gravitational force.	Since G < K, Gravitational force is lesser than Coulomb force.
4.	It depends on nature of the medium.	It is independent of the medium.
5.	Two point charges at rest experience only Coulomb force but they experience additional Lorentz force during at motion.	The gravitational force between two point masses is the same whether two masses are at rest or in motion.

13. State Superposition principle of multiple charges.

The total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.

$$i.e. \quad \vec{F}_1^{\,tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n}$$

14. Define an electric field. Give its unit.

The electric field at the point P at a distance r from the point charge q is defined as the force experienced by a unit charge placed at that point. Its unit is NC^{-1}.

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

15. What are the kinds of electric field?

- Uniform electric field.
- Non-uniform electric field.

16. What is an uniform electric field?

The electric field, which has the same direction and constant magnitude at all points in space, is called uniform electric field.

17.What is non-uniform electric field?

The electric field, which has different directions or different magnitudes or both at different points in space, is called non-uniform electric field.

18.What is superposition of electric fields?

The electric field at an arbitrary point due to a collection of point charges is simply equal to the vector sum of the electric fields created by the individual point charges. This is called superposition of electric fields.

$$i.e. \quad \vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots + \vec{E}_n$$

19.What is Linear charge density? Give its unit.

Charge per unit length is called linear charge density. Its unit is C m^{-1}.

$$i.e. \quad \lambda = \frac{Q}{L}$$

20.What is Surface charge density? Give its unit.

Charge per unit area is called surface charge density. Its unit is C m^{-2}.

$$i.e. \quad \sigma = \frac{Q}{A}$$

21.What is Volume charge density? Give its unit.

Charge per unit volume is called volume charge density. Its unit is C m^{-3}.

$$i.e. \quad \rho = \frac{Q}{V}$$

22.What is meant by 'Electric field lines'?

The electric field lines are visual representation of the electric field in some region of space.

23.What are the rules adopted for drawing the electric field lines of charges? (or) What are the properties of electric field lines?

- ❖ The electric field lines start from a positive charge and end at negative charges or at infinity.
- ❖ The electric field vector at a point in space is tangential to the electric field line at that point.
- ❖ The electric field lines are denser (more closer) in a region where the electric field has larger magnitude and less dense in a region where the electric field is of smaller magnitude. In other words, the number of lines passing through a given surface area perpendicular to the lines is proportional to the magnitude of the electric field in that region. *i.e.* $N \propto E \propto \frac{1}{r^2}$
- ❖ No two electric field lines intersect each other.
- ❖ The number of electric field lines that emanate from the positive charge or end at a negative charge is directly proportional to the magnitude of the charges. *i.e.* $N \propto q$

24.The electric field lines never intersect. Justify.

If two lines cross at a point, then there will be two different electric field vectors at the same point. Consequently, if some charge is placed in the intersection point, then it has to move in two different directions at the same time, which is physically impossible. Hence, electric field lines do not intersect.

25.What is an electric dipole?Give examples.

Two equal and opposite charges separated by a small distance constitute an electric dipole.

Ex: CO, water, ammonia, HCl etc.

26.What is an electric dipole moment? Give its unit.

The magnitude of the electric dipole moment is equal to the product of the magnitude of one of the charges and the distance between them. Its unit is C m.

$$i.e. \quad p = 2qa$$

27.Explain the principle and working of microwave oven.

- ❖ Microwave oven works on the principle of torque acting on an electric dipole.
- ❖ The food we consume has water molecules, which are permanent electric dipoles.
- ❖ Oven produces microwaves that are oscillating electromagnetic fields and produce torque on the water molecules.
- ❖ Due to this torque on each water molecule, the molecules rotate very fast and produce thermal energy. Thus, heat generated is used to heat the food..

28.Define electric potential difference.

The electric potential difference is defined as the work done by an external force to bring unit positive charge from point R to point P.

$$i.e. \quad V_P - V_R = \Delta V = -\int_R^P \vec{E} \cdot \overrightarrow{dr}$$

29. Define electric potential. Give its unit.

The electric potential at a point P is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point P in the region of the external electric field $\vec{E}$. Its unit is J C^{-1} or volt.

$$i.e. \quad V_P = -\int_{\infty}^P \vec{E} \cdot \overrightarrow{dr}$$

30.What is an equipotential surface?

An equipotential surface is a surface on which all the points are at the same potential.

31.What are the properties of an equipotential surface?

- The work done to move a charge q between any two points A and B in equipotential surface is zero. *i.e.* $W = q\,(V_B - V_A) = 0$ Since $V_A = V_B$ in equipotential surface.

- The electric field is always normal to an equipotential surface. If it is not normal, there will be work done to move a charge between two points due to parallel component of the electric field. This is contradictory to equipotential surface.

32.Give the relation between electric field and electric potential.

The electric field is the negative gradient of the electric potential.

$$i.e.\ \ E = -\frac{dv}{dx}$$

33.Define 'Electrostatic potential energy'.

The electrostatic potential energy is defined as the work done in arranging two point charges with a distance 'r' between them.

$$i.e.\ \ U = q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

34.What is electric flux?Give its unit.

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux. Its unit is N m^2 C^{-1}.

$$i.e.\ \ \Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta \qquad (uniform\ \vec{E})$$

$$\Phi_E = \int \vec{E}\cdot\vec{A} \qquad (non-uniform\ \vec{E})$$

$$\Phi_E = \oint \vec{E}\cdot\vec{A} \quad (closed\ \vec{A}\ \&\ non-uniform\ \vec{E})$$

35.State Gauss law.

Gauss law states that the total electric flux Φ_E through an arbitrary closed surface is $\frac{1}{\varepsilon_0}$ times the net charge Q enclosed in it.

$$i.e.\ \ \Phi_E = \frac{Q_{encl}}{\varepsilon_0}$$

36.What is meant by electrostatic shielding?

Certain part which is fully protected from the surrounding electric field is known as electrostatic shielding.

37.Why it is safer to sit inside a bus than in open ground or under a tree during the rain with thunder and lightning?

The metal body of the bus provides electrostatic shielding, since the electric field inside is zero. During lightning, the charges flow through the body of the conductor to the ground with no effect on the person inside that bus.

38.What is meant by electrostatic induction?

Charging a conductor without its actual contact is called electrostatic induction.

39.What is a dielectric?Give the examples

A dielectric is a non-conducting material and has no free electrons. The electrons in a dielectric are bound within the atoms.

Ex: Ebonite, glass and mica.

40.What is a non-polar molecule? Give the examples.

A non-polar molecule is one in which centers of positive and negative charges coincide. As a result, it has no permanent dipole moment.

Ex: H_2, O_2, CO_2.

41.What is a polar molecule? Give the examples.

A polar molecule is one in which the centers of the positive and negative charges are separated even in the absence of an external electric field. They have a permanent dipole moment.

Ex: H_2O, N_2O, HCl, NH_3.

42.Define Polarisation.

Polarisation $\vec{P}$ is defined as the total dipole moment per unit volume of the dielectric.

$$i.e.\ \ \vec{P} = \chi_e \vec{E}_{ext}\ \ ;\ \chi_e - \text{Electirc susceptibility.}$$

43.Define electric susceptibility.

The electric susceptibility is defined as the polarisation per unit external electric field. It has no unit.

44.What is dielectric breakdown?

When the external electric field applied to a dielectric is very large, it tears the atoms apart so that the bound charges become free charges. Then the dielectric starts to conduct electricity. This is called dielectric breakdown.

45.What is dielectric strength?

The maximum electric field the dielectric can withstand before it breakdowns is called dielectric strength. For example, the dielectric strength of air is 3×10^6 V m^{-1}.

46.What is a Capacitor?

Capacitor is a device used to store electric charge and electrical energy.

47.Define 'Capacitance of a capacitor'. Give its unit.

The capacitance of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between the conductors. Its unit is C V^{-1} or farad (F).

$$i.e.\ \ C = \frac{Q}{V}$$

In practice, capacitors are available in the range of microfarad ($1\mu F = 10^{-6}$ F) to picofarad($1pf = 10^{-12}$F).

48. Define ‘Electrostatic potential energy density’.

The Electrostatic potential energy stored per unit volume of space is defined as Electrostatic potential energy density.

$$i.e. \quad u_E = \frac{U}{V} = \frac{1}{2}\varepsilon_0 E^2$$

49. What are the applications or uses of capacitors?

- ❖ Flash capacitors are used in the digital camera to make flash light during the photo shoot.
- ❖ Capacitor of 175μF, 2000V is used in heart defibrillator which retrieves the normal heart function during cardiac arrest.
- ❖ Capacitors are used in the ignition system of automobile engines to eliminate sparking.
- ❖ Capacitors are used to reduce power fluctuations in power supplies and to increase the efficiency of power transmission.

50. What happens to charge, voltage, electric field, capacitance and energy stored in the capacitor when dielectric is inserted between the plates of a capacitor without and with battery connection?

Parameter	Dielectric is inserted when battery is disconnected	Dielectric is inserted when battery is connected
Charge	Constant	Increases
Voltage	Decreases	Constant
Electric field	Decreases	Constant
Capacitance	Increases	Increases
Energy stored	Decreases	Increases

51. What is action at points or Corona discharge?

The reduction of total charges of the conductor near the sharp edge is called action at points or Corona discharge.

5 Marks Q & A:

1. Discuss the basic properties of electric charges.

(a) Electric charge :

- ❖ Electric charge is the intrinsic and fundamental property of particles. The SI unit of charge is coulomb.

(b) Conservation of charges :

- ❖ The total electric charge in the universe is constant and charge can neither be created nor be destroyed. In any physical process, the net change in charge will always be zero. This is called conservation of charges.

(c) Quantisation of charges:

- ❖ The fundamental unit of charge is ‘e’. The charge q on any object is equal to an integral multiple of this fundamental unit of charge ‘e’.
- ❖ *i.e.* **q = ne**, Here n is any integer (0, ±1, ±2, ±3, ±4,...). This is called quantisation of electric charge.

2. Explain in detail Coulomb’s law and its various aspects.

- ❖ Coulomb's law states that the electrostatic force is directly proportional to the product of the magnitude of the two point charges and is inversely proportional to the square of the distance between those two point charges.
- ❖ The force on the charge q_2 exerted by the charge q_1 always lies along the line joining the two charges. $\hat{r}_{12}$ is the unit vector pointing from charge q_1 to q_2. Likewise, the force on the charge q_1 exerted by q_2 is along $-\hat{r}_{12}$ (i.e. along $\hat{r}_{21}$).
- ❖ In SI units, $k = \frac{1}{4\pi\varepsilon_0}$ and its value is $9 \times 10^9 Nm^2C^{-2}$. Here ε_o is the permittivity of free space or vacuum and the value of $\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$.
- ❖ The magnitude of the electrostatic force between two charges each of one coulomb and separated by a distance of 1 m is calculated as follows:

$$|F| = \frac{9 \times 10^9 \times 1 \times 1}{1^2} = 9 \times 10^9\ N$$

This is a huge quantity, almost equivalent to the weight of one million ton. We never come across 1 coulomb of charge in practice. Most of the electrical phenomena in day-to-day life involve electrical charges of the order of μC (micro coulomb) or nC (nano coulomb).

- In SI units, Coulomb's law in vacuum takes the form

$$\vec{F} = \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

For other medium $\varepsilon_o = \varepsilon$,

$$\vec{F}_m = \vec{F}_{21} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Since $\varepsilon > \varepsilon_o$, $\vec{F}_m < \vec{F}$. We define the relative permittivity for a given medium as $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$. For vacuum or air, $\varepsilon_r = 1$ and for all other media $\varepsilon_r > 1$.

- The force on a charge q_1 exerted by a point charge q_2 is given by

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

Similarly, the force on a charge q_2 exerted by a point charge q_1 is given by

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

But $\hat{r}_{21} = -\hat{r}_{12}$, Therefore, $\vec{F}_{12} = -\vec{F}_{21}$

- The expression for Coulomb force is true only for point charges. But the point charge is an ideal concept. However, we can apply Coulomb's law for two charged objects whose sizes are very much smaller than the distance between them. In fact, Coulomb discovered his law by considering the charged spheres in the torsion balance as point charges. The distance between the two charged spheres is much greater than the radii of the spheres.

3. Define 'Electric field' and discuss its various aspects.

- The electric field at the point P at a distance r from the point charge q is defined as the force experienced by a unit charge placed at that point. Its unit is NC^{-1}.

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

- If q is +ve, $\vec{E}$ points away from source charge q and if q is -ve, $\vec{E}$ points towards the charge q.

- The force experienced by the test charge q_0 placed at point P in the electric field $\vec{E}$ is $\vec{F} = q_0\vec{E}$.

- In $\vec{F} = q_0\vec{E}$, is $\vec{E}$ independent of q_0 and $\vec{E}$ depends only source charge q.

- If distance 'r' increases $\vec{E}$ decreases and $\vec{E}$ vanishes at infinity.

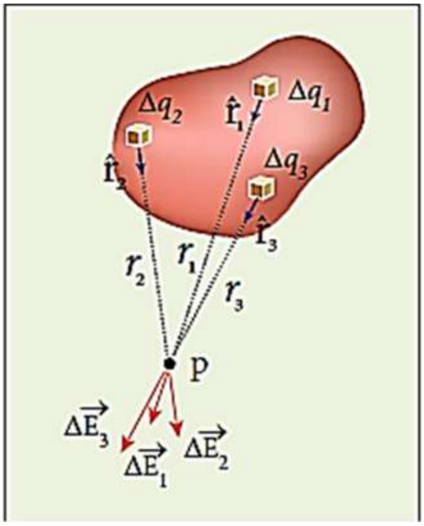
- In the definition of $\vec{E}$, test charge q_0 is taken sufficiently small. Such that q_0 will not change the position of source charge q and as well as its $\vec{E}$.

- $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ is valid for point charges(monopoles) For continuous and finite size charge distribution, integration techniques must be used. However, this expression can be approximated for a finite sized charge if the test point is far away from 'q'.

- There are two kinds of electric fields namely uniform and non-uniform electric fields.

4. How do we determine the electric field due to a continuous charge distribution? Explain.

- Consider the following charged object of irregular shape as shown in figure. The entire charged object is divided into a large number of charge elements $\Delta q_1, \Delta q_2, \Delta q_3, \ldots \Delta q_n$ and each charge element Δq is taken as a point charge.



- The electric field at a point P due to a charged object is approximately given by the sum of the fields at P due to all such charge elements.

$$\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\left[\frac{\Delta q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{\Delta q_2}{r_{2P}^2}\hat{r}_{2P} + \cdots + \frac{\Delta q_n}{r_{nP}^2}\hat{r}_{nP}\right]$$

$$\vec{E} \approx \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{\Delta q_i}{r_{iP}^2}\hat{r}_{iP} \longrightarrow (1)$$

- Here Δq_i is the i^{th} charge element, r_{iP} is the distance of the point P from the i^{th} charge element and $\hat{r}_{iP}$ is the unit vector from i^{th} charge element to the point P. However the equation (1) is only an approximation.

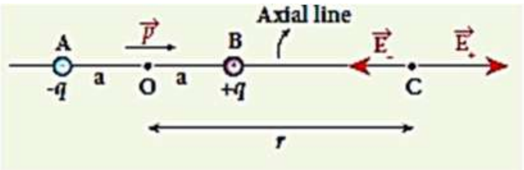
- To incorporate the continuous distribution of charge, we take the limit $\Delta q \to 0 (= dq)$. In this limit, the summation in the equation (1) becomes an integration and takes the following form

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$

- Here r is the distance of the point P from the infinitesimal charge dq and $\hat{r}$ is the unit vector from dq to point P.

5. Obtain an expression for electric field due to an electric dipole at points on the axial line.

- Consider an electric dipole placed on the x-axis as shown in Figure. A point C is located at a distance of r from the midpoint O of the dipole along the axial line.



- The electric field at C due to charge +q ,

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} \qquad (along\ BC\ \&\ \vec{p})$$

- The electric field at C due to charge -q ,

$$\vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p} \qquad (opp.to\ \vec{p})$$

- Since +q is located closer to the point C than –q, $\vec{E}_+$ is stronger than $\vec{E}_-$. Therefore, the length of the $\vec{E}_-$ vector is drawn larger than that of $\vec{E}_-$ vector.

- According to superposition principle, the total electric field at point C is calculated as,

$$\vec{E}_{tot} = \vec{E}_+ + \vec{E}_-$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p}$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r-a)^2} - \frac{q}{(r+a)^2}\right]\hat{p}$$

$$\vec{E}_{tot} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ra}{(r^2-a^2)^2}\right]\hat{p} \dashrightarrow (1)$$

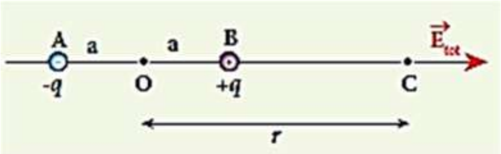
- If the point C is very far away from the dipole then (r >> a). Under this limit the term $(r^2-a^2)^2 \approx r^4$ Substituting this into equation (1), we get,

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left[\frac{4qa}{r^3}\right]\hat{p}$$

- Since $\vec{p} = 2aq\,\hat{p}$

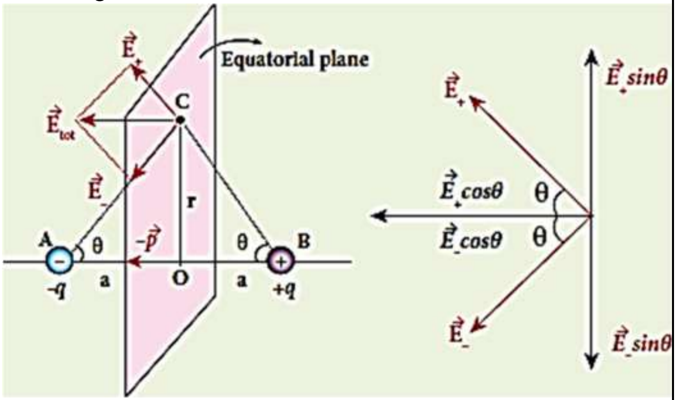
$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$

- This total electric field $\vec{E}_{tot}$ is along $\vec{E}_+$, since + q is closer to C than –q as shown in figure.



6. Obtain an expression for electric field due to an electric dipole at points on the equitorial line.

- Consider a point C at a distance r from the midpoint O of the dipole on the equatorial plane as shown in figure.



- Since the point C is equi-distant from +q and –q, the magnitude of the electric fields of +q and –q are the same. $i.e.\ |\vec{E}_+| = |\vec{E}_-|$

- The direction of $\vec{E}_+$ is along BC and the direction of $\vec{E}_-$ is along CA. $\vec{E}_+$ and $\vec{E}_-$ are resolved into parallel and perpendicular components.The perpendicular components $\vec{E}_+$sinθ and $\vec{E}_-$sinθ are oppositely directed and cancel each other.

- The magnitude of the total electric field $\vec{E}_{tot}$ at point C is the sum of the parallel components of $\vec{E}_+$ and $\vec{E}_-$ and its direction is along $-\vec{p}$ as shown in figure.

$$\vec{E}_{tot} = -\vec{E}_+ cos\theta\,\hat{p} - \vec{E}_- cos\theta\,\hat{p} \dashrightarrow (1)$$

- The magnitudes $\vec{E}_+$ and $\vec{E}_-$ are the same and are given by,

$$|\vec{E}_+| = |\vec{E}_-| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)} \dashrightarrow (2)$$

- Substituting equation(2) in (1), we get,

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{2q\cos\theta}{(r^2+a^2)}\hat{p}$$

- Since $cos\theta = \frac{a}{\sqrt{r^2+a^2}}$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{2aq}{(r^2+a^2)^{\frac{3}{2}}}\hat{p}$$

- Since $\vec{p} = 2aq\,\hat{p}$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^2+a^2)^{\frac{3}{2}}}$$

- At larger distance, r >> a, thus, $(r^2+a^2)^{\frac{3}{2}} \approx r^3$

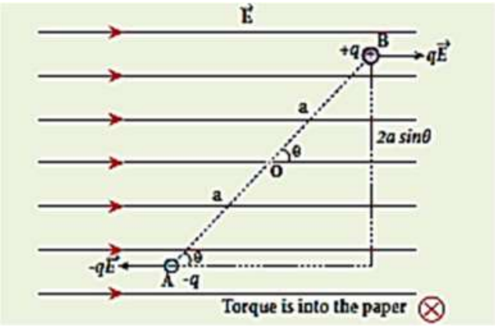
$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$$

Important Points about $\vec{E}_{dipole}$:

- ✓ $|\vec{E}_{axis}| = 2 \times |\vec{E}_{equatorial}| \qquad (at\ r \gg a)$
- ✓ $\vec{E}_{axis}$ is along $\vec{p}$ and $\vec{E}_{equatorial}$ is opposite to $\vec{p}$
- ✓ $\vec{E}_{dipole} \propto \frac{1}{r^3}\ (at\ r \gg a)$, but $\vec{E}_{monopole} \propto \frac{1}{r^2}$. This implies that at r >> a, $\vec{E}_{dipole}$ goes to zero faster than $\vec{E}_{monopole}$. It means, at r >> a, the two charges in dipole appear closer and neutralize each other.
- ✓ $\vec{E}_{axis} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$ and $\vec{E}_{equatorial} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$ both are valid at r >> a. Suppose $2a \to 0$ & $q \to \infty$, p =2aq will be finite. Then the dipole is called point dipole. For such point dipoles, the equations $\vec{E}_{axis}$ and $\vec{E}_{equatorial}$ are exact and valid for any ‘r’.

7. Obtain an expression for the torque experienced by an electric dipole in the uniform electric field.

- ❖ Consider an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$ whose field lines are equally spaced and point in the same direction.
- ❖ The charge +q will experience a force q$\vec{E}$ in the direction of the field and charge –q will experience a force -q$\vec{E}$ in a direction opposite to the field.
- ❖ Since the external field $\vec{E}$ is uniform, the total force acting on the dipole is zero. These two forces acting at different points will constitute a couple and the dipole experience a torque as shown in figure.



- ❖ This torque tends to rotate the dipole. (Note that electric field lines of a uniform field are equally spaced and point in the same direction). The total torque on the dipole about the point O,

$$\vec{\tau} = -\overrightarrow{OA} \times q\vec{E} + \overrightarrow{OB} \times q\vec{E}$$

- ❖ Using right-hand corkscrew rule, it is found that total torque is perpendicular to the plane of the paper and is directed into it.
- ❖ The magnitude of the total torque is given by,

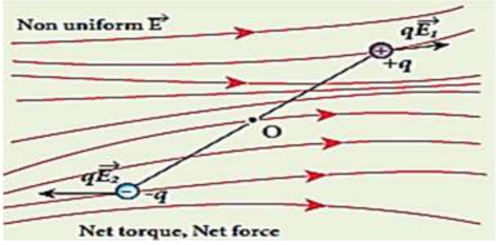
$$|\vec{\tau}| = |\overrightarrow{OA}|\,|-q\vec{E}|sin\theta + |\overrightarrow{OB}||q\vec{E}|sin\theta$$

$$\tau = a\,qE\,sin\theta + a\,qE\,sin\theta$$

$$\tau = 2aq\,E\,sin\theta$$

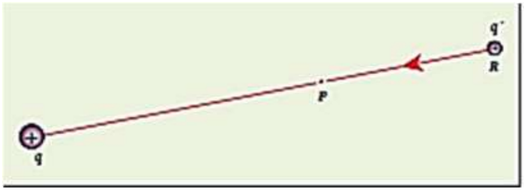
$$\tau = pE\,sin\theta \qquad [\because\ \ p = 2a\,q]$$

- ❖ Where θ is the angle made by $\vec{p}$ with $\vec{E}$. At θ = 90^0, Torque τ = pE (maximum) and at θ =0^0 τ = 0.
- ❖ In vector form, $\vec{\tau} = \vec{p} \times \vec{E}$
- ❖ If the electric field is not uniform, there will be net force acting on the dipole in addition to the torque as shown in figure.



8. Obtain a relation between electrostatic potential energy and electrostatic potential.

- ❖ Consider a positive charge q kept fixed at the origin, which produces an electricfield $\vec{E}$ around it.
- ❖ A positive test charge q′ is brought from point R to point P with external force against the repulsive force between q and q′ $(\vec{F}_{ext} = -\vec{F}_{coulomb})$ as shown in figure.



- ❖ Work must be done to overcome this repulsion. This work done is stored as potential energy. The work done is

$$w = \int_R^P \vec{F}_{ext} \cdot \overrightarrow{dr}$$

- ❖ Since coulomb force is conservative, work done is independent of the path and it depends only on the initial and final positions of the test charge.
- ❖ If potential energy associated with q′ at P is U_P and thatat R is U_R, then difference in potential energyis defined as the work done to bring a test charge q′ from point R to P and is given as $U_P - U_R$ = W = ΔU

$$\Delta U = \int_R^P \vec{F}_{ext} \cdot \overrightarrow{dr}$$

- ❖ Since $\vec{F}_{ext} = -\vec{F}_{coulomb} = -q'\vec{E}$

$$\Delta U = \int_R^P \left(-q'\vec{E}\right) \cdot \overrightarrow{dr} = q' \int_R^P \left(-\vec{E}\right) \cdot \overrightarrow{dr}$$

$$\frac{\Delta U}{q'} = -\int_R^P \vec{E}\cdot\vec{dr}$$

- The above equation is independent of q' and it is equal to potential difference between P an R. Therefore,

$$V_P - V_R = \Delta V = \frac{\Delta U}{q'} = -\int_R^P \vec{E}\cdot\vec{dr}$$

- The electric potential energy difference can be written as

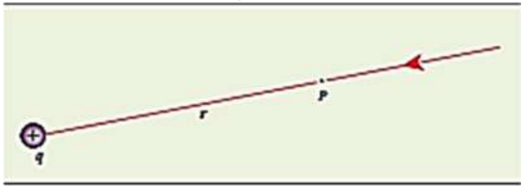
$$\Delta U = q'\,\Delta V$$

- If R = ∞, V_∞ = 0. Therefore, The electric potential at P is

$$V_P = -\int_\infty^P \vec{E}\cdot\vec{dr}$$

9. Obtain an expression for the potential due to a point charge.

- Consider a positive charge q kept fixed at the origin. Let P be a point at distance r from the charge q. This is shown in Figure.



- The electric potential at the point P is

$$V = \int_\infty^r (-\vec{E})\cdot\vec{dr} = -\int_\infty^r \vec{E}\cdot\vec{dr}$$

- Electric field due to positive point charge q is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

- Therefore,

$$V = \frac{-1}{4\pi\varepsilon_0}\int_\infty^r \frac{q}{r^2}\hat{r}\cdot\vec{dr}$$

- The infinitesimal displacement vector, $\vec{dr} = dr\,\hat{r}$ and using $\hat{r}\cdot\hat{r} = 1$, we have

$$V = \frac{-1}{4\pi\varepsilon_0}\int_\infty^r \frac{q}{r^2}\hat{r}\cdot dr\,\hat{r} = \frac{-1}{4\pi\varepsilon_0}\int_\infty^r \frac{q}{r^2}dr$$

- After the integration,

$$V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_\infty^r = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

- Hence the electric potential due to a point charge q at a distance r is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

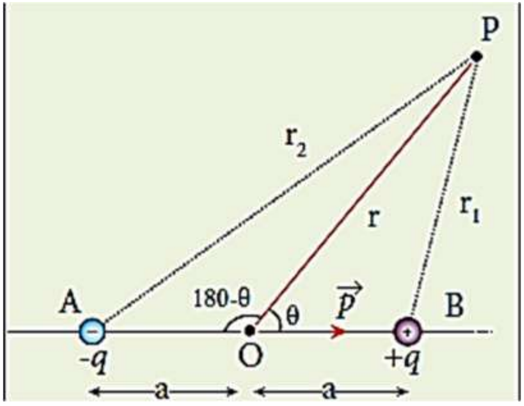
Important Points :

- ✓ If q is +ve, V>0 and if q is –ve, V is also -ve.
- ✓ The description of motion of objects using the concept of potential or potential energy is simpler than that using the concept of field.
- ✓ $V_+ \downarrow$ as $r \uparrow$ whereas $V_- \uparrow$ as $r \uparrow$. V = 0 at r = ∞.
- ✓ +q moves from $V_{high} \to V_{low}$ but -q moves from $V_{low} \to V_{high}$.
- ✓ $V_P = V_{q1} + V_{q2} + V_{q3} + \cdots + V_{qn}$

$$V_P = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \cdots + \frac{kq_n}{r_n} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

10. Obtain an electrostatic potential at a point due to an electric dipole. Discuss the special cases.

- Consider two equal and opposite charges separated by a small distance 2a as shown in figure. The point P is located at a distance r from the midpoint of the dipole. Let θ be the angle between the line OP and dipole axis AB.



- Let r_1 be the distance of point P from +q and r_2 be the distance of point P from –q.

- Potential at P due to +q,

$$V_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1}$$

- Potential at P due to -q,

$$V_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r_2}$$

- Total potential at the point P,

$$V = V_+ + V_-$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \dashrightarrow (1)$$

- By the cosine law for triangle BOP,

$$r_1^2 = r^2 + a^2 - 2ra\cos\theta$$

$$r_1^2 = r^2\left[1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right]$$

- Since the point P is very far from dipole, then r>>a. As a result the term $\frac{a^2}{r^2}$ is very small and can be neglected. Therefore,

$$r_1^2 = r^2\left[1-\frac{2a}{r}cos\theta\right]$$
$$r_1 = r\left[1-\frac{2a}{r}cos\theta\right]^{\frac{1}{2}}$$
$$\frac{1}{r_1} = \frac{1}{r}\left[1-\frac{2a}{r}cos\theta\right]^{-\frac{1}{2}}$$

- Since $\frac{a}{r}$<<1, we can use binomial theorem and retain the terms up to first order

$$\frac{1}{r_1} = \frac{1}{r}\left[1+\frac{1}{2}\cdot\frac{2a}{r}cos\theta\right]$$
$$\frac{1}{r_1} = \frac{1}{r}\left[1+\frac{a}{r}cos\theta\right] \dashrightarrow (2)$$

- Similarly applying the cosine law for triangle AOP,

$$r_2^2 = r^2 + a^2 - 2ra\,cos(180-\theta)$$

- Since $cos(180-\theta) = -cos\theta$, we get,

$$r_2^2 = r^2 + a^2 + 2ra\,cos\theta$$
$$r_2^2 = r^2\left[1+\frac{a^2}{r^2}+\frac{2a}{r}cos\theta\right]$$

- Neglecting the term $\frac{a^2}{r^2}$ (because r>>a)

$$r_2^2 = r^2\left[1+\frac{2a}{r}cos\theta\right]$$
$$r_2 = r\left[1+\frac{2a}{r}cos\theta\right]^{\frac{1}{2}}$$
$$\frac{1}{r_2} = \frac{1}{r}\left[1+\frac{2a}{r}cos\theta\right]^{-\frac{1}{2}}$$

- Using Binomial theorem, we get

$$\frac{1}{r_2} = \frac{1}{r}\left[1-\frac{1}{2}\cdot\frac{2a}{r}cos\theta\right]$$
$$\frac{1}{r_2} = \frac{1}{r}\left[1-\frac{a}{r}cos\theta\right] \dashrightarrow (3)$$

- Substituting equation(2) and (3) in equation(1),

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left[1+\frac{a}{r}cos\theta\right]-\frac{1}{r}\left[1-\frac{a}{r}cos\theta\right]\right]$$
$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left[1+\frac{a}{r}cos\theta-1+\frac{a}{r}cos\theta\right]$$
$$V = \frac{1}{4\pi\varepsilon_0}\frac{2aq}{r^2}cos\theta$$

- But the electric dipole moment p = 2aq and we get,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{pcos\theta}{r^2}\right]$$

- Now we can write $p\,cos\theta = \vec{p}.\hat{r}$, where $\hat{r}$ is the unit vector from the point O to point P. Hence the electric potential at a point P due to an electric dipole is given by,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p}.\hat{r}}{r^2}\right] \dashrightarrow (4)$$

- Equation (4) is valid for distances very large compared to the size of the dipole(i.e. r>>a). But for a point dipole, the equation (4) is valid for any distance.

- **Special cases :**

Case (i): If the point P lies on the axial line of the dipole on the side of +q, then θ = 0^0. Then the electric potential becomes

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{p}{r^2}\right]$$

Case (ii): If the point P lies on the axial line of the dipole on the side of –q, then θ = 180^0, then

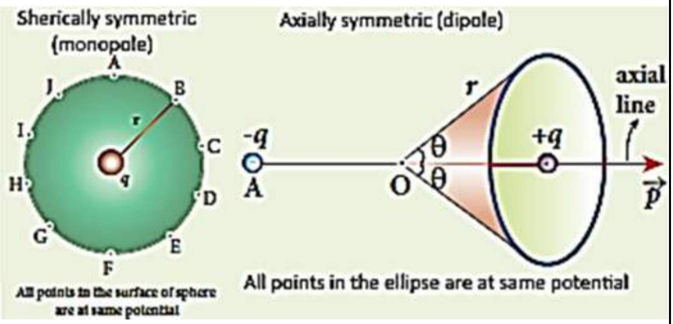
$$V = -\frac{1}{4\pi\varepsilon_0}\left[\frac{p}{r^2}\right]$$

Case (iii): If the point P lies on the equatorial line of the dipole, then θ = 90^0. Hence $V = 0.$

Important Points :

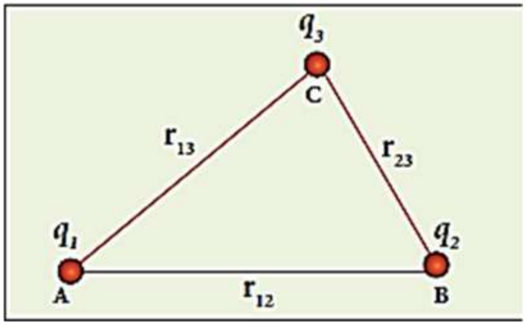
- ✓ $V_{dipole} \propto \frac{1}{r^2}$ and $V_{monopole} \propto \frac{1}{r}$. Therefore, V_{dipole} falls faster than $V_{monoploe}$. If ‘r’ increases +ve and –ve charges in dipole nullify each other.

- ✓ $V_{monopole}$ is spherically symmetric and it depends only on ‘r’. But V_{dipole} is axially symmetric and it depends on ‘θ’(angle b/w $\vec{p}$ & $\hat{r}$).



1. Obtain an expression for potential energy due to a collection of three point charges, which are separated by finite distances.

- Three charges are arranged in the following configuration as shown in figure.



- To calculate the total electrostatic potential energy, we use the following procedure. We bring all the charges one by one and arrange them according to the configuration as shown in above figure.

- First, bringing a charge q_1 from infinity to the point 'A' requires no work, because there are no other charges already present near charge q_1.

$$i.e. \quad W_1 = U_1 = 0$$

- Next to bring the second charge q_2 to the point B, work must be done against the electric field due to the charge q_1. If V_{1B} is the electrostatic potential due to the charge q_1 at point B, the work done on the charge q_2 is

$$W_2 = U_2 = q_2 V_{1B}$$

But $V_{1B} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}}$, Therefore potential energy,

$$U_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}}\right]$$

- Similarly, to bring the charge q_3 to the point C, work has to be done against the total electric field due to both charges q_1 and q_2. So the work done to bring the charge q_3 is

$$W_3 = U_3 = q_3 \left(V_{1C} + V_{2C}\right)$$

Here V_{1C} and V_{2C} are the electrostatic potentials at point C due to q_1 and q_2 respectively.

- But $V_{1C} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}}$, $V_{2C} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}$, Therefore potential energy,

$$U_3 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

- The total electrostatic potential energy for the system of three charges q_1, q_2 and q_3 is

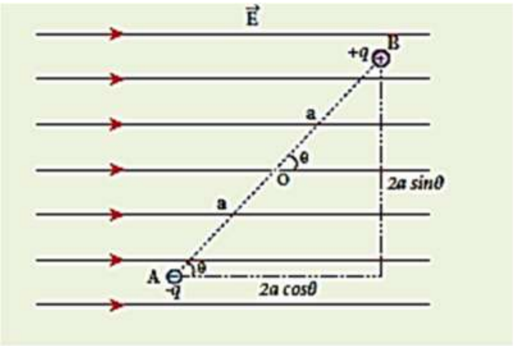
$$U = U_1 + U_2 + U_3$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right] \dashrightarrow (1)$$

- The expression (1) is same if the charges are aligned in any order, since the Coulomb force involved here is a conservative force.

2. Derive an expression for electrostatic potential energy of a dipole in a uniform electric field.

- Consider a dipole placed in the uniform electric field $\vec{E}$ as shown in the figure.



- A dipole experiences a torque when kept in an uniform electric field $\vec{E}$. This torque rotates the dipole to align it with the direction of the electric field.

- To rotate the dipole (at constant angular velocity) from its initial angle θ′ to another angle θ against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole.

- The work done by the external torque to rotate the dipole from angle θ′ to θ at constant angular velocity is

$$W = \int_{\theta'}^{\theta} \tau_{ext}\, d\theta \dashrightarrow (1)$$

- Since $\vec{\tau}_{ext}$ is equal and opposite to $\vec{\tau}_E = \vec{p} \times \vec{E}$, we have,

$$\tau_{ext} = |\vec{\tau}_{ext}| = |\vec{\tau}_E| = |\vec{p} \times \vec{E}| = pE\, sin\theta$$

- Substituting this in equation (1), we get,

$$W = \int_{\theta'}^{\theta} pE\, sin\theta\, d\theta = pE[-cos\theta]_{\theta'}^{\theta}$$

$$W = pE(-\cos\theta + cos\theta')$$

- This work done is equal to the potential energy difference between the angular positions θ and θ′.

$$U(\theta) - U(\theta') = W = -pEcos\theta + pE\cos\theta'$$

- If the initial angle is θ′ = 90^0 and is taken as reference point, then $U(\theta') = pE\cos 90^0 = 0$.

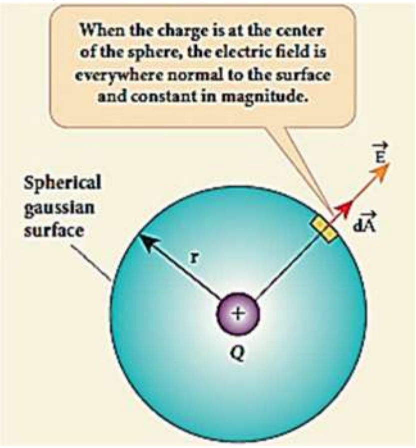
- The potential energy stored in the system of dipole kept in the uniform electric field is given by

$$U = U(\theta) = -pE\cos\theta = \vec{p}\cdot\vec{E}$$

- U depends on p, E and θ (angle b/w $\vec{p}$ & $\vec{E}$). If θ = π, U is maximum and if θ = 0, U is minimum.

⑬ Obtain Gauss law from Coulomb's law.

- A positive point charge Q is surrounded by an imaginary sphere of radius r as shown in figure.


When the charge is at the center of the sphere, the electric field is everywhere normal to the surface and constant in magnitude.

- We can calculate the total electric flux through the closed surface of the sphere as,

$$\Phi_E = \oint \vec{E} \cdot \overrightarrow{dA} = \oint EdA\, cos\theta$$

- The electric field of the point charge is directed radially outward at all points on the surface of the sphere. Therefore, the direction of the area element $\overrightarrow{dA}$ is along the electric field $\vec{E}$ and θ= 0^0.

$$\Phi_E = \oint EdA \qquad [\,Since\; cos0^0 = 1]$$

$$\Phi_E = E\oint dA$$

- Since $\oint dA = 4\pi r^2$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ (From Coulomb's law), we have

$$\Phi_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \times 4\pi r^2$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

- The equation is called as Gauss's law. This is equally true for any arbitrary shaped surface, which encloses the charge $Q_{encl.}$ So we can write,

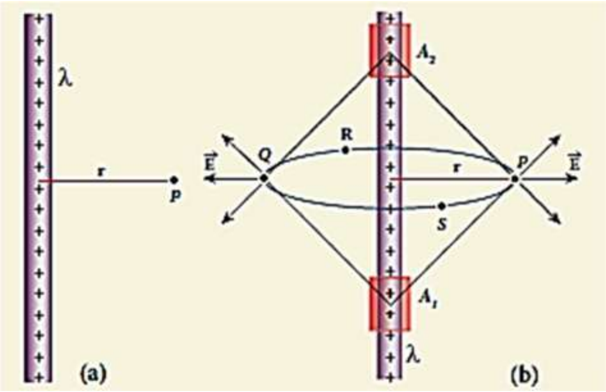
$$\Phi_E = \oint \vec{E} \cdot \overrightarrow{dA} = \frac{Q_{encl}}{\varepsilon_0}$$

Important Points about Gauss law :

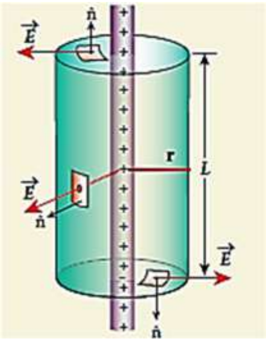
- ✓ Gauss law is $\Phi_E = \frac{Q_{encl}}{\varepsilon_0}$. Here total flux Φ_E depends only on the charge enclosed by the surface and not depends the charge outside the surface.
- ✓ Φ_E is independent of location of charges enclosed by the surface.
- ✓ The shape of the Gaussian surface depends on the type of charge configuration and its symmetry. There are spherical, planar and cylindrical Gaussian surfaces.
- ✓ Though Q_{encl} denotes only inside charges, the term electric field 'E' is due to the charges present inside and outside the Gaussian surface.
- ✓ The Gaussian surface can pass through continuous charge distribution and cannot pass through discrete charge distribution because here 'E' is not well defined.
- ✓ Gauss law is another form of Coulomb's law but it is also applicable for charges in motion. So it is more general than Coulomb's law.

⑭ Arrive at an expression for electric field due to an infinitely long charged wire using Gauss law.

- Consider an infinitely long straight wire having uniform linear charge density λ. Let P be a point located at a perpendicular distance r from the wire (Figure (a)).



- The electric field at the point P can be found using Gauss law. We choose two small charge elements A_1 and A_2 on the wire, which are at equal distances from the point P.
- The resultant electric field due to A_1 and A_2 points radially outwards and the magnitude of electric field is same at all points on the circle of radius r as shown in fig.(b). Hence, we can infer that the charged wire possesses a cylindrical symmetry.



- Let us choose a cylindrical Gaussian surface of radius r and length L as shown in the figure above.
- The total electric flux in this closed surface is calculated as follows

$$\Phi_E = \oint \vec{E} \cdot \overrightarrow{dA}$$

$$\Phi_E = \int_{\substack{Curved \\ surface}} \vec{E}\cdot\overrightarrow{dA} + \int_{\substack{top \\ surface}} \vec{E}\cdot\overrightarrow{dA} + \int_{\substack{bottom \\ surface}} \vec{E}\cdot\overrightarrow{dA}$$

$$\Phi_E = \int_{\substack{Curved \\ surface}} EdA\,cos\theta + \int_{\substack{top \\ surface}} EdA\,cos\theta + \int_{\substack{bottom \\ surface}} EdA\,cos\theta$$

-->(1)

- From fig. at curved surface $\vec{E}$ is parallel to $\vec{A}$

  $$i.e. \quad EdA\,cos0^0 = E\,dA$$

  and at top & bottom $\vec{E}$ is perpendicular to $\vec{A}$

  $$i.e. \quad EdA\,cos90^0 = 0$$

- Substituting these values in equation(1) and applying Gauss law,

$$\Phi_E = \int_{\substack{Curved \\ surface}} EdA = \frac{Q_{encl}}{\varepsilon_0}$$

- Since $\vec{E}$ is constant in entire curved surface, it is taken outside the integration and if 'λ' is linear charge density of the charged wire, $Q_{encl} = \lambda L$.

$$E\int_{\substack{Curved \\ surface}} dA = \frac{\lambda L}{\varepsilon_0}$$

  Here total area of the curved surface, $\int_{\substack{Curved \\ surface}} dA = 2\pi rL$.

- Therefore,

$$E \cdot 2\pi rL = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

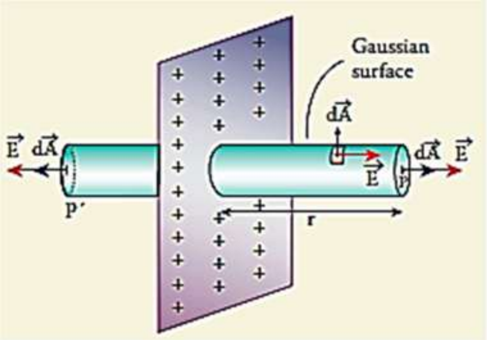
- In vector form, $\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\ \hat{r} \dashrightarrow (2)$

Important Points:

- ✓ Here $\vec{E} \propto \frac{1}{r}$ whereas in point charge $\vec{E} \propto \frac{1}{r^2}$.
- ✓ $\vec{E}\ \perp wire$. If λ >0 , $\vec{E}$ points perpendicular outward from wire $(\hat{r})$ and if λ<0 , $\vec{E}$ points perpendicular inward$(-\hat{r})$.
- ✓ For finite length charged wire, it is true at mid point and at points far away from both ends of the wire, where only $\vec{E}$ is approximately radial.

15. Arrive at an expression for electric field due to charged infinite plane sheet using Gauss law.

- Consider an infinite plane sheet of charges with uniform surface charge density σ. Let P be a point at a distance of r from the sheet as shown in the figure.



- Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed at all points.
- A cylindrical shaped Gaussian surface of length 2r and area A of the flat surfaces is chosen such that the infinite plane sheet passes perpendicularly through the middle part of the Gaussian surface.
- Applying Gauss law for this cylindrical surface,

$$\Phi_E = \int_{\substack{Curved \\ surface}} \vec{E}\cdot\overrightarrow{dA} + \int_P \vec{E}\cdot\overrightarrow{dA} + \int_{P'} \vec{E}\cdot\overrightarrow{dA} = \frac{Q_{encl}}{\varepsilon_0}$$

- From fig. $\vec{E}$ is perpendicular to $\overrightarrow{dA}$ at curved surface and is parallel to $\vec{A}$ at P & P'. Hence,

$$\Phi_E = \int_P E\cdot dA + \int_{P'} E\cdot dA = \frac{Q_{encl}}{\varepsilon_0}$$

- Since $\vec{E}$ is uniform at surfaces P and P', it is taken outside the integration and if 'σ' is surface charge density of the charged sheet, $Q_{encl} = \sigma A$.

$$E\int_P dA + E\int_{P'} dA = \frac{\sigma A}{\varepsilon_0}$$

- The total area of the surface at P and P' are equal to 'A'.

$$i.e. \quad \int_P dA = \int_{P'} dA = A$$

- Therefore,

$$EA + EA = \frac{\sigma A}{\varepsilon_0}$$

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

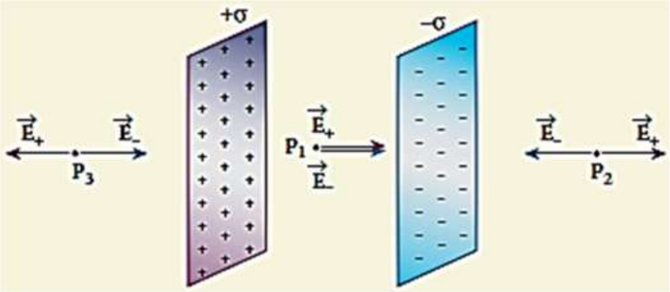
- In vector form, $\vec{E} = \frac{\sigma}{2\varepsilon_0}\ \hat{n} \rightarrow (1)$. Here $\hat{n}$ is the outward unit vector normal to the plane.

Important Points:

- ✓ Here $\vec{E}$ depends on 'σ' and is independent of distance 'r'.
- ✓ $\vec{E}$ is same at any point farther away from the charged plane.
- ✓ If σ >0, $\vec{E}$ points outward perpendicular to the plane $(\hat{n})$ and if σ < 0 , $\vec{E}$ points inward perpendicular $(-\hat{r})$ to the plane.
- ✓ For a finite charged plane sheet, equation (1.71) is approximately true only in the middle region of the plane and at points far away from both ends.

16.Arrive at an expression for electric field due to two parallel charged infinite sheets using Gauss law.

- ❖ Consider two infinitely large charged plane sheets with equal and opposite charge densities +σ and -σ which are placed parallel to each other as shown in the figure.



- ❖ The electric field between the plates and outside the plates is found using Gauss law. The magnitude of the electric field due to an infinite charged plane sheet is $\frac{\sigma}{2\varepsilon_0}$ and it points outward normal if σ > 0 and points inward normal if σ < 0.
- ❖ At the points P$_2$ and P$_3$, the electric field due to both plates are equal in magnitude and opposite in direction (Figure). As a result, electric field at a point outside the plates is zero.

$$i.e.\ E_{outside} = 0$$

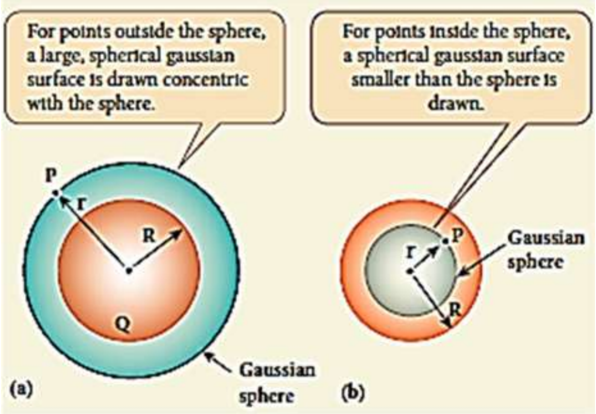
- ❖ But inside the plates, electric fields are in same direction i.e., towards the right, the total electric field at a point P$_1$

$$E_{inside} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

- ❖ The direction of the electric field inside the plates is directed from positively charged plate to negatively charged plate and is uniform everywhere inside the plates.

✓17.Arrive at an expression for Electric field due to a uniformly charged spherical shell using Gauss law.

- ❖ Consider a uniformly charged spherical shell of radius R and total charge Q as shown in figure.
- ❖ The electric field at points outside and inside the sphere can be found using Gauss law.



Case (a) : **At a point outside the shell (r > R)**

- ❖ Let us choose a point P outside the shell at a distance r from the center as shown in figure(a).
- ❖ The charge is uniformly distributed on the surface of the sphere (spherical symmetry). Hence the electric field must point radially outward if Q > 0 and point radially inward if Q < 0.
- ❖ So we choose a spherical Gaussian surface of radius r is chosen and the total charge enclosed by this Gaussian surface is Q.
- ❖ Applying Gauss law

$$\oint_{\substack{Gaussian \\ surface}} \vec{E} \cdot \overrightarrow{dA} = \frac{Q}{\varepsilon_0}$$

- ❖ Here $\vec{E}$ is perpendicular to $\overrightarrow{dA}$. Since $\vec{E}$ is uniform at all points on the spherical Gaussian surface, 'E' is taken outside the integration.

$$E \oint_{\substack{Gaussian \\ surface}} dA = \frac{Q}{\varepsilon_0}$$

- ❖ But $\oint_{\substack{Gaussian \\ surface}} dA = 4\pi r^2$, the total area of the spherical Gaussian surface.

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

- ❖ In vector form, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,\hat{r}$. Here $\vec{E}$ is radially outward if Q>0 and is radially inward if Q<0. This $\vec{E}$ will be same as if the entire charge Q is concentrated at the center of the spherical shell.

Case (b) : **At a point on the surface of the spherical shell (r = R)**

- ❖ The electrical field at points on the spherical shell (r = R) is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\,\hat{r}$$

Case (c) : **At a point inside the spherical shell (r < R)**

- Consider a point P inside the shell at a distance r from the center. A Gaussian sphere of radius r is constructed as shown in the figure(b).
- Applying Gauss law

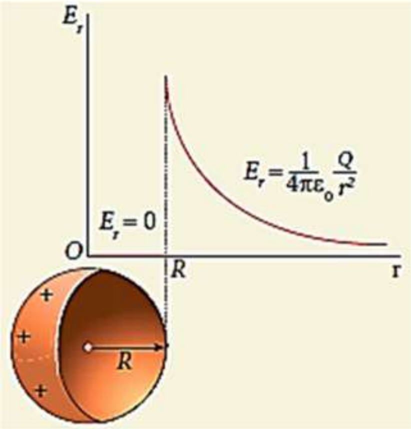
$$\oint_{\substack{Gaussian\\surface}} \vec{E}\cdot\overrightarrow{dA} = \frac{Q}{\varepsilon_0}$$

$$E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \longrightarrow (1)$$

- Since Gaussian surface encloses no charge, So Q = 0. The equation (1) becomes

$$E = 0 \qquad (r < R)$$

- The electric field due to the uniformly charged spherical shell is zero at all points inside the shell.
- A graph is plotted between the electric field and radial distance. This is shown in figure below.



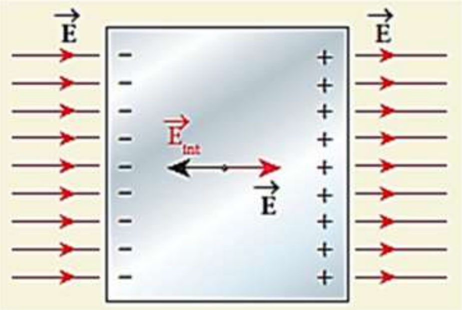
18.Discuss the various properties of conductors in electrostatic equilibrium.

- An electrical conductor has a large number of free electrons. When there is no external electric field, they are in continuous random motion in all directions at the surface. As a result, there is no net motion of electrons (or net current) along any direction. Now the conductor is at electrostatic equilibrium.

A conductor at electrostatic equilibrium has the following properties.

(i) The electric field is zero everywhere inside the conductor. This is true regardless of whether the conductor is solid or hollow.

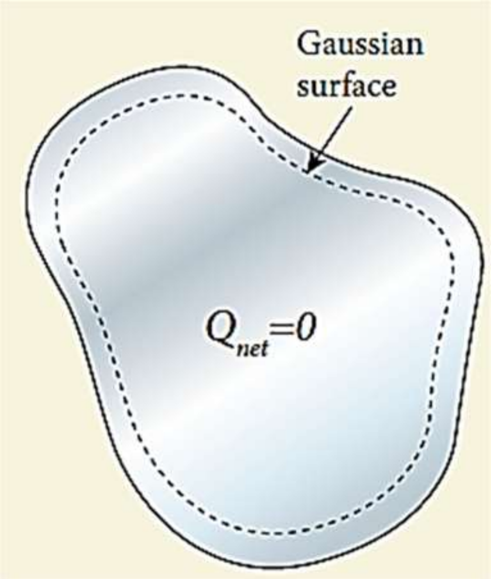
- When an external electric field $\vec{E}$ is applied from left to right of the conductor, the negative charges are acummulated at the left and positive charges are at right. This inturn develops and increases the opposite internal electric field $\vec{E}_{int}$ until it nullifies the external electric field $\vec{E}$ as shown in figure below.



- Once the external electric field is nullified the conductor is said to be in electrostatic equilibrium. The time taken for this is in the order of 10^{-16}s, which can be taken as almost instantaneous.

(ii) There is no net charge inside the conductors. The charges must reside only on the surface of the conductors.

- We can prove this property using Gauss law. Consider an arbitrarily shaped conductor as shown in figure.

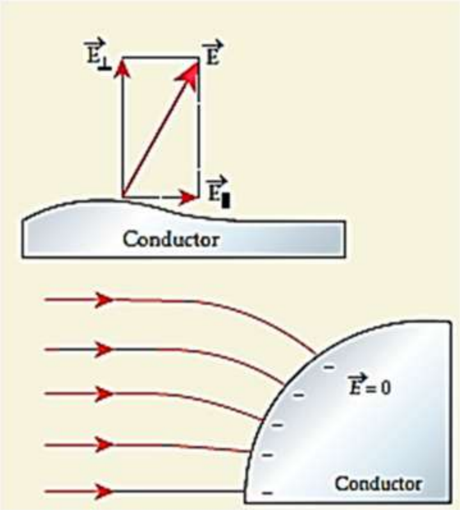


- A Gaussian surface is drawn inside the conductor such that it is very close to the surface of the conductor. Since the electric field is zero everywhere inside the conductor, the net electric flux is also zero over this Gaussian surface.
- From Gauss's law, this implies that there is no net charge inside the conductor. Even if some charge is introduced inside the conductor, it immediately reaches the surface of the conductor.

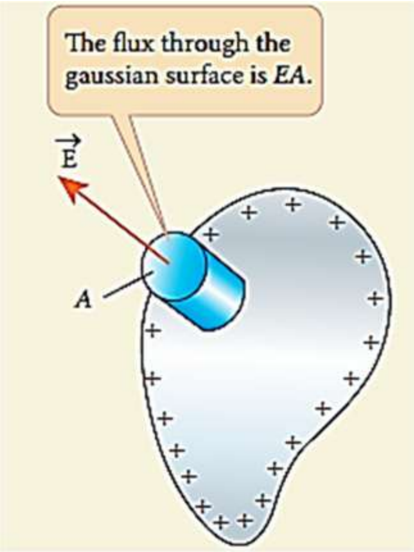
(iii) The electric field outside the conductor is perpendicular to the surface of the conductor and has a magnitude of $\frac{\sigma}{\varepsilon_0}$ where σ is the surface charge density at that point.

- If the electric field has components parallel to the surface of the conductor, then free electrons on the surface of the conductor would experience acceleration (Figure (a)). This means that the conductor is not in equilibrium, which is contradicted to the concept.

- Therefore, at electrostatic equilibrium, the electric field must be perpendicular to the surface of the conductor. This is shown in Figure (b).



- We now prove that the electric field has magnitude $\frac{\sigma}{\varepsilon_0}$ just outside the conductor's surface. Consider a small cylindrical Gaussian surface, as shown in the figure below. One-half of this cylinder is embedded inside the conductor.


The flux through the gaussian surface is *EA*.

- Since electric field is normal to the surface of the conductor, the curved part of the cylinder has zero electric flux. Also inside the conductor, the electric field is zero. Hence the bottom flat part of the Gaussian surface has no electric flux.

- Therefore, the top flat surface alone contributes to the electric flux. The electric field is parallel to the area vector and the total charge inside this top flat surface is σA. By applying Gauss's law,

$$EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

In vector form, $\vec{E} = \frac{\sigma}{\varepsilon_0}\,\hat{n}$

- Here $\hat{n}$ represents the unit vector outward normal to the surface of the conductor. Suppose $\sigma < 0$, then the electric field acts inward perpendicular to the surface.

(iv)The electrostatic potential has the same value on the surface and inside of the conductor.

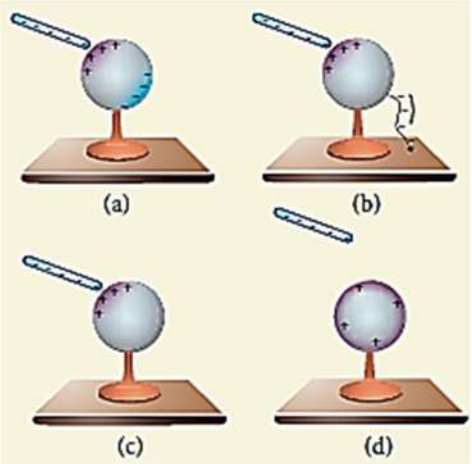
- We know that the conductor has no parallel electric component on the surface, which means that charges can be moved on the surface without doing any work. This is possible only if the electrostatic potential is constant at all points on the surface and there is no potential difference between any two points on the surface.

- Since the electric field is zero inside the conductor, the potential is the same as the surface of the conductor. Thus at electrostatic equilibrium, the conductor is always at equipotential.

19.Explain the process of electrostatic induction.

- Charging an object without its actual contact is called electrostatic induction.

- Consider an uncharged (neutral) conducting sphere at rest on an insulating stand. Suppose a negatively charged rod is brought near the conductor without touching it, as shown in figure(a).



- Before introducing the charged rod, the free electrons were distributed uniformly on the surface of the conductor and the net charge is zero.

- Once the negatively charged rod is brought near the conductor, the distribution is no longer uniform with more electrons located on the farther side of the rod and positive charges are located closer to the rod. But the total charge is zero.

- Now the conducting sphere is connected to the ground through a conducting wire. This is called grounding. Since the ground can always receive any amount of electrons, grounding removes the electron from the conducting sphere. Note that positive charges will not flow to the ground because they are attracted by the negative charges of the rod (Figure (b)).

- ❖ When the grounding wire is removed from the conductor, the positive charges remain near the charged rod (Figure (c))

- ❖ Now the charged rod is taken away from the conductor. As soon as the charged rod is removed, the positive charge gets distributed uniformly on the surface of the conductor (Figure (d)). By this process, the neutral conducting sphere becomes positively charged.

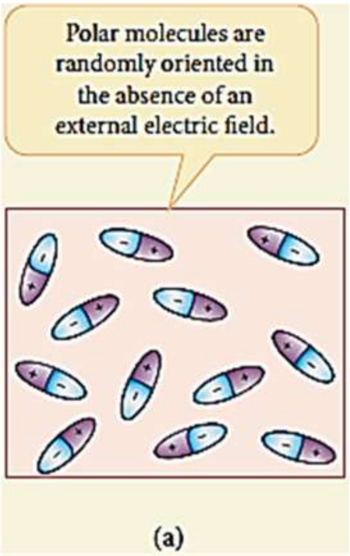
- ❖ For an arbitrary shaped conductor, the intermediate steps and conclusion are the same except the final step. The distribution of positive charges is not uniform for arbitrarily shaped conductors.

20. Explain dielectrics in detail and how an electric field is induced inside a dielectric.

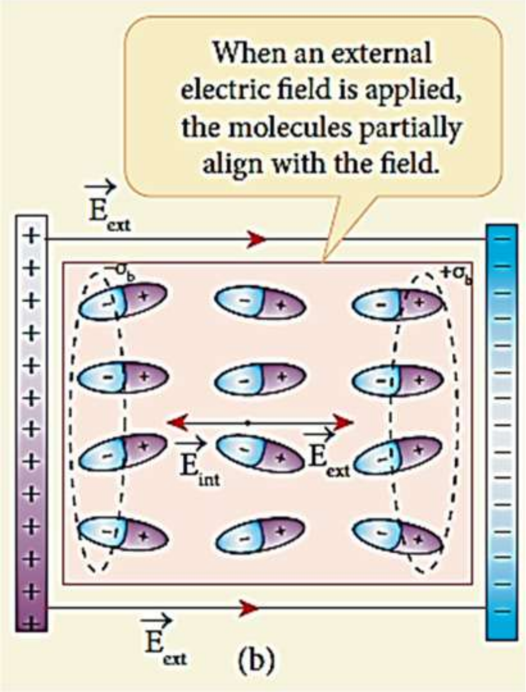
- ❖ When an external electric field is applied on a conductor, the charges are aligned in such a way that an internal electric field is created which cancels the external electric field.

- ❖ But in the case of a dielectric, which has no free electrons, the external electric field only realigns the charges so that an internal electric field is produced.

- ❖ The magnitude of the internal electric field is smaller than that of external electric field. Therefore, the net electric field inside the dielectric is not zero but is anti-parallel to an external electric field with magnitude less than that of the external electric field.



- ❖ Let us consider a rectangular dielectric slab placed between two oppositely charged plates (capacitor) as shown in the figure (b).



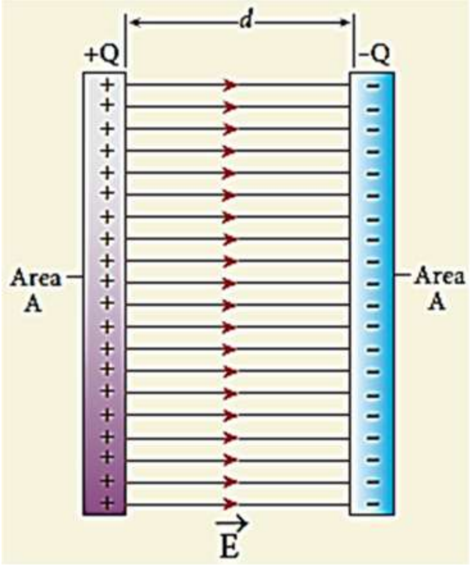
- ❖ The uniform electric field between the plates acts as an external electric field $\vec{E}_{ext}$, which polarizes the dielectric placed between plates.

- ❖ The positive charges are induced on one side surface and negative charges are induced on the other side of surface. But inside the dielectric, the net charge is zero even in a small volume.

- ❖ So the dielectric in the external field is equivalent to two oppositely charged sheets with the surface charge densities $+\sigma_b$ and $-\sigma_b$. These charges are called bound charges. They are not free to move like free electrons in conductors. This is shown in the figure (b).

21. Obtain the expression for capacitance for a parallel plate capacitor.

- ❖ Consider a capacitor with two parallel plates each of cross-sectional area 'A' and separated by a distance 'd' as shown in figure.



- The electric field between two infinite parallel plates is uniform and is given by

$$E = \frac{\sigma}{\varepsilon_0}$$

where σ is the surface charge density on the plates

- If the separation distance d is very much smaller than the size of the plate (d << A), then the above result is used even for finite–sized parallel plate capacitor.

- The electric field between the plates is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \qquad \left(\because\ \sigma = \frac{Q}{A}\right)$$

- Since the electric field is uniform, the electric potential between the plates having separation 'd' is given by

$$V = Ed = \frac{Qd}{A\varepsilon_0}$$

- Therefore the capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)} = \frac{\varepsilon_0 A}{d}$$

- From the above equation, it can be seen that capacitance is directly proportional to the area of cross section and is inversely proportional to the distance between the plates.

Important Points:

- ✓ If 'A' is increased, more charges can be distributed for the same potential difference.So C is increased.
- ✓ If 'd' decreases, $V_{Capacitor} < V_{battery}$. Now Charges flow from battery to capacitor till $V_{Capacitor} = V_{battery}$.
- ✓ If 'd' increases, $V_{Capacitor} > V_{battery}$. Now Charges flow from capacitor to battery till $V_{Capacitor} = V_{battery}$.

22) Obtain the expression for energy stored in the parallel plate capacitor.

- Capacitor not only stores the charge but also it stores energy. When a battery is connected to the capacitor, electrons of total charge -Q are transferred from one plate to the other plate. To transfer the charge, work is done by the battery. This work done is stored as electrostatic potential energy in the capacitor.

- To transfer an infinitesimal charge dQ for a potential difference V, the work done is given by

$$dW = V\,dQ$$

$$dW = \frac{Q}{C}dQ \qquad \left[\because\ V = \frac{Q}{C}\right]$$

- The total work done to charge a capacitor is

$$W = \int_0^Q \frac{Q}{C}dQ = \frac{Q^2}{2C}$$

- This work done is stored as electrostatic potential energy (U_E) in the capacitor.

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}CV^2 \qquad [\because\ Q = CV]$$

- This stored energy is thus directly proportional to the capacitance of the capacitor and the square of the voltage between the plates of the capacitor.

- By using $C = \frac{\varepsilon_0 A}{d}$ and V = Ed, we get,

$$U_E = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2$$

Where Ad is the volume of the space between the capacitor plates.

- The energy stored per unit volume(energy density) of space is defined as energy density,

$$u_E = \frac{U_E}{Volume} = \frac{U_E}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

- From the above equation, we infer that the energy is stored in the electric field existing between the plates of the capacitor. Once the capacitor is allowed to discharge, the energy is retrieved.

- It is important to note that the energy density depends only on the electric field and not on the size of the plates of the capacitor.

23) Explain in detail the effect of a dielectric placed in a parallel plate capacitor.

The dielectric can be inserted into the plates in two different ways.

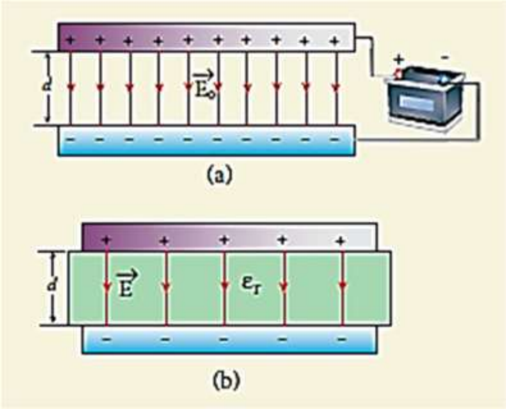
(i) When the capacitor is disconnected from the battery:

- Consider a capacitor with two parallel plates each of cross-sectional area A and are separated by a distance d. The capacitor is charged by a battery of voltage V_0 and the charge stored is Q_0.

- The capacitance of the capacitor without the dielectric is,

$$C_0 = \frac{Q_0}{V_0}$$

- The battery is then disconnected from the capacitor and the dielectric is inserted between the plates. This is shown in figure.



- The introduction of dielectric between the plates will decrease the electric field, which is given by,

$$E = \frac{E_0}{\varepsilon_r}$$

Here E_o is the electric field inside the capacitors when there is no dielectric and ε_r is the relative permittivity of the dielectric or simply known as the dielectric constant.

- Since $\varepsilon_r > 1$, the electric field $E < E_o$. As a result, the electrostatic potential difference between the plates ($V = Ed$) is also reduced. But the charge Q_o is constant once the battery is disconnected.
- Hence the new potential difference is,

$$V = Ed = \frac{E_0}{\varepsilon_r} d = \frac{V_0}{\varepsilon_r}$$

- Since C is inversely proportional to V, when V decreases C increases. Thus new capacitance in the presence of a dielectric is,

$$C = \frac{Q_0}{V} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0$$

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon A}{d}$$

Here $C_0 = \frac{\varepsilon_0 A}{d}$ and $\varepsilon = \varepsilon_r \varepsilon_0$, the permittivity of the dielectric.

- Since $\varepsilon_r > 1$, we have $C > C_o$. Thus the insertion of the dielectric increases the capacitance.

- The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{Q_0^2}{2C_0}$$

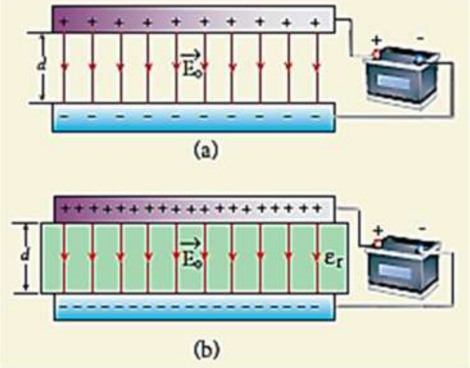
- After the dielectric is inserted, the charge Q_0 remains constant but the capacitance is increased. As a result, the stored energy is decreased.

$$U = \frac{Q_0^2}{2C} = \frac{Q_0^2}{2\varepsilon_r C_0} = \frac{U_0}{\varepsilon_r}$$

- Since $\varepsilon_r > 1$ we get $U < U_o$. There is a decrease in energy because, when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.

(ii) When the battery remains connected to the capacitor:

- Consider the capacitor with dielectric inserted is connected to the battery of voltage V_0 as shown in figure.



- When dielectric is inserted, the potential difference V_0 across the plates remains constant. But experimentally it is found that the charge stored in the capacitor is increased by a factor ε_r.

$$Q = \varepsilon_r Q_0$$

- Due to this increased charge, the capacitance is also increased. The new capacitance is

$$C = \frac{Q}{V_0} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0$$

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon A}{d}$$

Here $C_0 = \frac{\varepsilon_0 A}{d}$ and $\varepsilon = \varepsilon_r \varepsilon_0$, the permittivity of the dielectric.

- The energy stored in the capacitor before the insertion of a dielectric is given by

$$U_0 = \frac{1}{2} C_0 V_0^2$$

- After the dielectric is inserted, the capacitance is increased. Hence, the stored energy is also increased.

$$U = \frac{1}{2} C V_0^2 = \frac{1}{2} \varepsilon_r C_0 V_0^2 = \varepsilon_r U_0$$

Since $\varepsilon_r > 1$ we have $U > U_o$.

- It may be noted here that since voltage between the capacitor V_0 is constant, the electric field between the plates also remains constant.

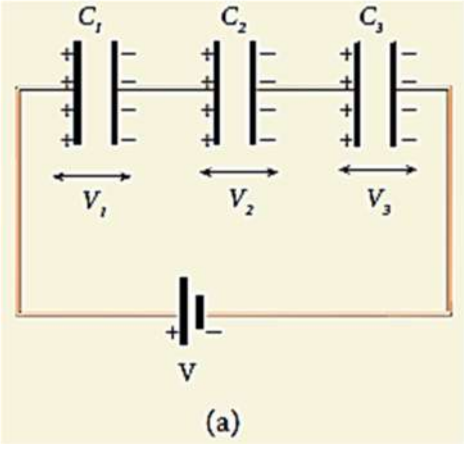
- The energy density is given by,

$$u = \frac{1}{2} \varepsilon E_0^2$$

24. Derive an expression for resultant capacitance, when capacitors are connected in series and in parallel.

(i) Capacitor in series:

- Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series with a battery of voltage V as shown in the figure(a).



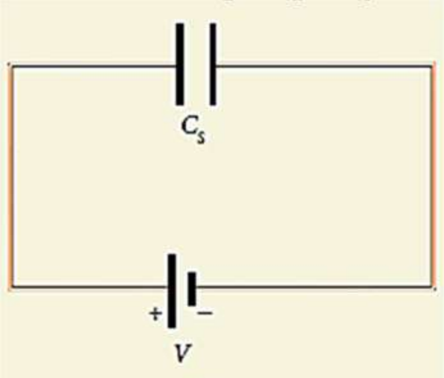
- As soon as the battery is connected to the capacitors in series, the electrons of charge –Q is transferred from the negative terminal to positive terminal via C_1, C_2 and C_3.

- In this process, each capacitor stores the same amount of charge Q. But the voltage across each capacitor is also different and are denoted as V_1, V_2 and V_3 respectively.
- The total voltage across each capacitor must be equal to the voltage of the battery.

$$V = V_1 + V_2 + V_3$$

- Since , $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$ & $V_3 = \frac{Q}{C_3}$ we have,

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \dashrightarrow (1)$$



(b)

- If three capacitors in series are considered to form an equivalent single capacitor C_s shown in figure(b), then we have

$$V = \frac{Q}{C_S}$$

Substituting this in equation(1), We get,

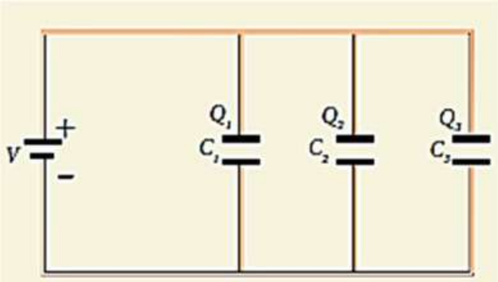
$$\frac{Q}{C_S} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- Thus, the inverse of the equivalent capacitance C_S of three capacitors connected in series is equal to the sum of the inverses of each capacitance.
- This equivalent capacitance C_S is always less than the smallest individual capacitance in the series.

(ii) Capacitance in parallel:

- Consider three capacitors of capacitance C_1, C_2 and C_3 connected in parallel with a battery of voltage V as shown in figure(a).



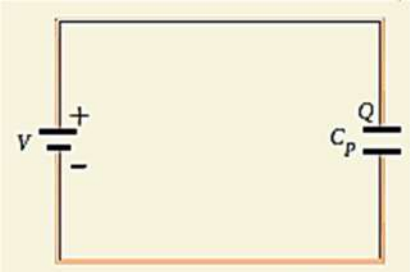
(a)

- Since corresponding sides of the capacitors are connected to the same positive and negative terminals of the battery, the voltage across each capacitor is equal to the battery's voltage.
- Since capacitance of the capacitors is different, the charge stored in each capacitor is not the same. Let the charge stored in the three capacitors be Q_1, Q_2, and Q_3 respectively.
- According to the law of conservation of total charge, the sum of these three charges is equal to the charge Q transferred by the battery,

$$Q = Q_1 + Q_2 + Q_3$$

- Now since $Q_1 = C_1V$, $Q_2 = C_2V$ & $Q_3 = C_3V$ we have,

$$Q = C_1V + C_2V + C_3V \dashrightarrow (1)$$



(b)

- If these three capacitors are considered to form a single capacitance C_P which stores the total charge Q as shown in the figure(b), then we can write

$$Q = C_PV$$

- Substituting this in equation(1), we get,

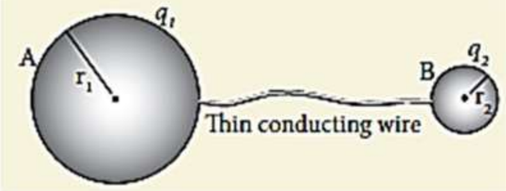
$$C_PV = C_1V + C_2V + C_3V$$

$$C_P = C_1 + C_2 + C_3$$

- Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitances.
- The equivalent capacitance C_P in a parallel connection is always greater than the largest individual capacitance.
- In a parallel connection, area of each capacitance adds to give more effective area such that total capacitance increases.

25) Explain in detail how charges are distributed in a conductor, and the principle behind the lightning conductor.

- Consider two conducting spheres A and B of radii r_1 and r_2 respectively connected to each other by a thin conducting wire as shown in the figure. The distance between the spheres is much greater than the radii of either spheres.



- If a charge Q is introduced into any one of the spheres, this charge Q is redistributed into both the spheres such that the electrostatic potential is same in both the spheres. They are now uniformly charged and attain electrostatic equilibrium.

- Let q_1 be the charge residing on the surface of sphere A and q_2 is the charge residing on the surface of sphere B such that Q = q1 + q2. The charges are distributed only on the surface and there is no net charge inside the conductor.

- The electrostatic potential at the surface of the sphere A is given by,

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

- The electrostatic potential at the surface of the sphere A is given by,

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

- Since the spheres are connected by the conducting wire, the surfaces of both the spheres together form an equipotential surface. This implies that

$$V_A = V_B$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \dashrightarrow (1)$$

- Let us take the charge density on the surface of sphere A is σ_1 and charge density on the surface of sphere B is σ_2. This implies that

$$q_1 = 4\pi r_1^2\sigma_1 \quad and$$
$$q_2 = 4\pi r_2^2\sigma_2$$

- Substituting theses values in equation(1), we get,

$$\sigma_1 r_1 = \sigma_2 r_2$$

- From which we conclude that

$$\sigma r = constant$$

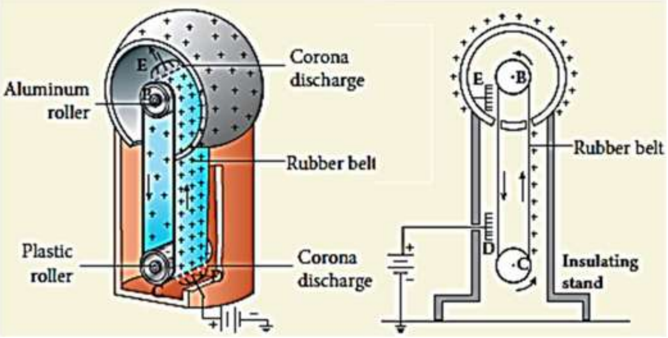
- Thus the surface charge density σ is inversely proportional to the radius of the sphere. For a smaller radius, the charge density will be larger and vice versa.

26) Explain in detail the construction and working of a Van de Graaff generator.

- Van de Graaff generator is a machine, which produces a large amount of electrostatic potential difference, up to several million volts (10^7 V).

- This Van de Graff generator works on the principle of electrostatic induction and action at points.

- A large hollow spherical conductor is fixed on the insulating stand as shown in figure.



- A pulley B is mounted at the center of the hollow sphere and another pulley C is fixed at the bottom.

- A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor.

- Two comb shaped metallic conductors E and D are fixed near the pulleys. The comb D is maintained at a positive potential of 10^4 V by a power supply.

- The upper comb E is connected to the inner side of the hollow metal sphere.

- Due to the high electric field near comb D, air between the belt and comb D gets ionized.

- The positive charges are pushed towards the belt and negative charges are attracted towards the comb D.

- The positive charges stick to the belt and move up. When the positive charges reach the comb E, a large amount of negative and positive charges are induced on either side of comb E due to electrostatic induction.

- As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere.

- Since the sphere is a conductor, the positive charges are distributed uniformly on the outer surface of the hollow sphere.

- At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.

- When the belt descends, it has almost no net charge. At the bottom, it again gains a large positive charge.

- The belt goes up and delivers the positive charges to the outer surface of the sphere.

- This process continues until the outer surface produces the potential difference of the order of 10^7 V, which is the limiting value.

- We cannot store charges beyond this limit since the extra charge starts leaking to the surroundings due to ionization of air.

- The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.

- The high voltage produced in this Van de Graaff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications.

2. Current Electricity

1. What is electric current? Give its unit.

The electric current in a conductor is defined as the rate of flow of charges through a given cross-sectional area A. Its unit is ampere (A). It is a scalar quantity.

$$I = \frac{Q}{t}$$

$$Average\ current, I_{avg} = \frac{\Delta Q}{\Delta t}$$

$$Instantaneous\ current, I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

2. Why current is a scalar?

Although current has a specific direction and magnitude, it does not obey for the law of vector addition. So that it is a scalar quantity.

3. What is meant by drift velocity ($\vec{V}_d$)? Give its unit.

The drift velocity is the average velocity acquired by the electrons inside the conductor when it is subjected to an electric field. Its unit is m s^{-1}.

4. Define mobility. Give its unit.

Mobility is defined as the magnitude of the drift velocity per unit electric field. Its unit is m^2 V^{-1} s^{-1}.

$$\mu = \frac{|\vec{v}_d|}{|\vec{E}|}$$

5. Distinguish between drift velocity and mobility.

S. No.	Drift velocity	Mobility
1.	The average velocity acquired by the electrons inside the conductor when it is subjected to an electric field.	The magnitude of the drift velocity per unit electric field.
2.	Its unit is m s^{-1}.	Its unit is m^2 V^{-1} s^{-1}.
3.	It is directly proportional to electric field.	It is constant for particular material of the conductor.

6. What is mean free time?

The average time between successive collisions is called the mean free time(τ).

7. Define current density($\vec{J}$). Give its unit.

The current density (J) is defined as the current per unit area of cross section of the conductor. Its unit is A m^{-2}. It is a vector quantity.

$$\vec{J} = \frac{I}{\vec{A}} = ne\vec{v}_d = \sigma\vec{E}$$

8. State microscopic form of Ohm's law.

The current density of the conductor is directly proportional to the applied electric field.

$$\vec{J} = \sigma\vec{E}$$

9. State macroscopic form of Ohm's law.

The current flowing through the conductor is directly proportional to the potential difference between the ends of the conductor.

$$V = IR$$

10. What are ohmic and non ohmic devices?

A device which obeys ohm's law is called ohmic device. <u>Ex:</u> wire, resistor,etc.

A device which do not obey ohm's law is called non-ohmic device.<u>Ex:</u> thermistors, crystal rectifiers, vacuum tube etc.

11. What is the resistance (R) ? Give its unit.

The resistance is the ratio of potential difference across the given conductor to the current passing through the conductor. Its unit is ohm (Ω).

$$R = \frac{V}{I}$$

12. Define electrical resistivity (ρ). Give its unit.

The electrical resistivity of a material is defined as the resistance offered to current flow by a conductor of unit length having unit area of cross section. Its unit is ohm-metre (Ω m).

$$\rho = \frac{RA}{l}$$

13. What is conductivity? Give its unit.

The reciprocal of resistivity is called conductivity. Its unit is ohm^{-1} m^{-1}.

$$\sigma = \frac{1}{\rho}$$

14. Define temperature coefficient of resistivity. Give its unit.

Temperature coefficient of resistivity is defined as the ratio of increase in resistivity per degree rise in temperature to its resistivity at T_o. Its unit is per ^{0}C.

$$\alpha = \frac{\rho_T - \rho_0}{\rho_0(T - T_0)} = \frac{\Delta\rho}{\rho_0\Delta T}$$

$$or \qquad \alpha = \frac{R_T - R_0}{R_0(T - T_0)} = \frac{\Delta R}{R_0\Delta T}$$

Where,

ρ_t , R_t – Resistivity and resistance at T^0 C respectively.

ρ_0, R_0 – Resistivity and resistance at T_0 ^{0} C respectively.

- **For conductors**,the resistivity increases with increase in temperature. So α is positive.
- **For Semiconductors**,the resistivity decreases with increase in temperature. So α is negative.

15. What is thermistor?

A semiconductor with a negative temperature coefficient of resistance is called a thermistor.

16. What is superconductivity?

The resistance of certain materials become zero below certain temperature. This property of material is known as superconductivity. The materials which exhibit this property are known as superconductors.

17. What is critical or transition temperature?

The temperature at which the normal conductor becomes superconductor is called critical temperature or transition temperature(T_c). [T_c of Hg = 4.2K]

18. What is electric power? Give its unit.

The electrical power P is the rate at which the electrical potential energy is delivered. Its unit is watt (W).

$$P = \frac{dU}{dt} = VI = I^2R = \frac{V^2}{R}$$

19. Define 1 watt.

1 watt is defined as electrical potential energy of 1 joule delivered per second. i.e. $1\ W = 1\ J\ s^{-1}$.

20. What is an electric energy?

An electrical energy is the work done by the moving streams of the electrons or charges.

- $Electrical\ energy = Electric\ Power\ \times time$
- The practical unit of electrical energy is kilowatt hour(**kWh**).
- 1 kWh = 1unit = 1000 Wh = 3.6 X 10^6 J.

21. Derive the expression for power P=VI.

The electric power is defined as,

$$P = \frac{dU}{dt} = \frac{d}{dt}(VQ) = V\frac{dQ}{dt} \qquad [\because U = VQ]$$

$$P = VI \qquad \left[\because I = \frac{dQ}{dt}\right]$$

22. What is an electric cell or battery?

An electric cell or a battery is a device, which converts chemical energy into electrical energy to produce electricity.

23. What is an electromotive force (emf) of a battery?

The emf of a battery or a cell is the voltage provided by the battery when no current flows in the external circuit. It is denoted by the symbol ξ (xi).

24. What do you mean by internal resistance of a cell?

The resistance offered by the electrolyte to the flow of charges within the battery is called internal resistance of the battery.

25. State Kirchhoff's first rule (Current or Junction rule).

The algebraic sum of the currents at any junction of a circuit is zero. (***It obeys for the law of conservation of charges.***)

26. State Kirchhoff's Second rule (Voltage or Loop rule).

In a closed circuit, the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit. (***It obeys for the law of conservation of energy of an isolated system.***)

27. What is Potentiometer?

Potentiometer is a device, which is used for the accurate measurement of potential differences, current and resistances.

28. State the principle of potentiometer.

The emf of the cell is directly proportional to the balancing length of the potetiometer wire. i.e. $\xi \alpha l$.

29. What is Joule's heating effect?

When current flows through a resistor, some of the electrical energy delivered to the resistor is converted into heat energy and it is dissipated. This heating effect of current is known as Joule's heating effect.

30. State Joule's law of heating.

The heat developed in an electrical circuit due to the flow of current varies directly as

(i) the square of the current
(ii) the resistance of the circuit and
(iii) the time of flow.

31. What is thermoelectric effect?

Conversion of temperature differences into electrical voltage and vice versa is known as thermoelectric effect.

32. What is Seebeck effect?

When the junctions of two dissimilar metals in a closed circuit are maintained at different temperatures an emf (potential difference) is developed. This is called Seebeck effect.

33. What is thermocouple?

The two dissimilar metals connected to form two junctions is known as thermocouple.

34. What are the applications of Seebeck effect?

- ❖ Seebeck effect is used in thermoelectric generators (Seebeck generators) which converts waste heat into electricity in power plants.
- ❖ It is utilized in automobiles as automotive thermoelectric generators for increasing fuel efficiency.
- ❖ It is used in thermocouples and thermopiles to measure the temperature difference between the two objects.

35. What is Peltier effect?

When an electric current is passed through a circuit of a thermocouple, heat is evolved at one junction and absorbed at the other junction. This is known as Peltier effect. It is reversible.

36. What is Thomson effect?

If two points in a conductor are at different temperatures, the density of electrons at these points will differ and as a result the potential difference is created between these points. This effect is called Thomson effect. It is reversible.

37. What is positive Thomson effect?

In some metals, heat is transferred due to the current flow in the direction of the current. It is called positive Thomson effect. Ex: Cu, Ag, zn, and cd.

38. What is negative Thomson effect?

In some metals, heat is transferred due to the current flow in the direction opposite to the current. It is called negative Thomson effect. Ex: Fe, pt, Ni, Co, and Hg.

5 Marks Q & A:

1. Derive an expresson for drift velocity.

- The acceleration $\vec{a}$ experienced by the electron in an electric field $\vec{E}$ is given by,

$$\vec{a} = \frac{-e\vec{E}}{m} \qquad [\because F = -eE]$$

- The drift velocity $\vec{v}_d$ is given by,

$$\vec{v}_d = \vec{a}\tau$$

$$\vec{v}_d = \frac{-e\tau}{m}\vec{E}$$

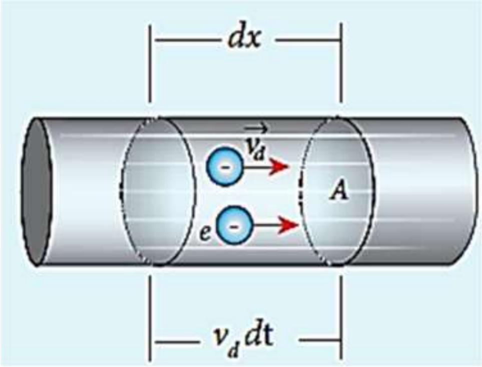
$$\vec{v}_d = -\mu\vec{E}$$

- Here $\mu = \frac{e\tau}{m}$ is the mobility of the electron.

$$\mu = \frac{|\vec{v}_d|}{|\vec{E}|}$$

2. Describe the microscopic model of current and obtain general form of Ohm's law.

- Consider a conductor with area of cross section A and an electric field $\vec{E}$ applied from right to left. Suppose there are n electrons per unit volume in the conductor, each moves with the same drift velocity $\vec{v}_d$ as shown in Figure.



- The distance travelled by the electrons within a small time interval dt,

$$dx = v_d dt \qquad \left[\because \quad v_d = \frac{dx}{dt}\right]$$

- No. of electrons available in the volume Adx is,

$$= Adx \times n$$

Substituting dx value,

$$= Av_d dt \times n$$

- Total charge in volume element(Adx) is,

$$dQ = \begin{pmatrix} electron \\ charge \end{pmatrix} \times \begin{pmatrix} no.of\ electrons\ in \\ the\ volume\ element(Adx) \end{pmatrix}$$

$$dQ = (e) \times Av_d dt\, n$$

- Hence the current,

$$I = \frac{dQ}{dt} = \frac{neAv_d dt}{dt}$$

$$I = neAv_d$$

- Now Current density is defined as,

$$J = \frac{I}{A}$$

- Substituting the value of I, we get,

$$J = \frac{neAv_d}{A}$$

$$J = nev_d$$

- In vector form,

$$\vec{J} = ne\vec{v}_d$$

$$\vec{J} = -\frac{ne^2\tau}{m}\vec{E} \qquad \left[\because \ \vec{v}_d = \frac{-e\tau}{m}\vec{E}\right]$$

$$\vec{J} = -\sigma\vec{E}$$

Where $\sigma = \frac{ne^2\tau}{m}$ is called conductivity.

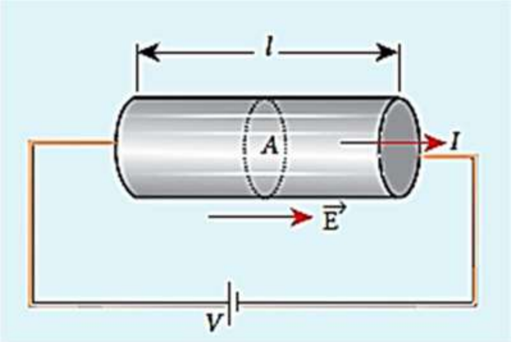
- But conventionally, we take direction of $\vec{J}$ is along $\vec{E}$. So that,

$$\vec{J} = \sigma\vec{E}$$

- This equation is called microscopic form of Ohm's law.

3. Obtain the macroscopic form of Ohm's law from its microscopic form and discuss its limitation.

- Consider a segment of wire of length l and cross sectional area A as shown in Figure.



- For simplicity, When a potential difference V is applied across the wire, a net uniform electric field is created in the wire which constitutes the current.

$$V = El$$

$$E = \frac{V}{l}$$

- We know the magnitude of current density,

$$J = \sigma E = \sigma\frac{V}{l}$$

- But $J = \frac{I}{A}$. Thus,

$$\frac{I}{A} = \sigma\frac{V}{l}$$

$$V = I\left(\frac{l}{\sigma A}\right)$$

$$V = IR$$

Where $R = \frac{l}{\sigma A}$ is called resistance of the conductor.

- Therefore, the macroscopic form of ohm's law can be stated as,

$$V = IR$$

Limitations of Ohm's law:

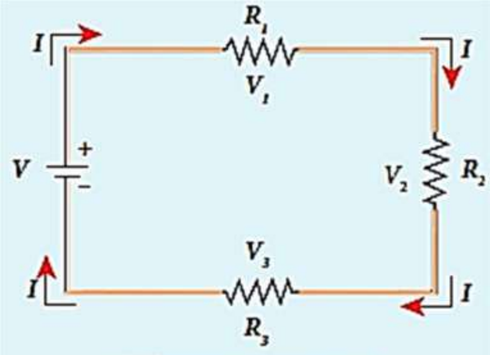
- According to Ohm's law $\frac{V}{I} = Constant$. This constant is R. But when temperature varies, R is not constant. Hence Ohm's law is not valid here.

- In some materials like diode, V is not not directly proportional to I. They are called non-ohmic devices. Here also Ohm's law is invalid.

4. Explain the equivalent resistance of a series and parallel resistor network.

(a) Resistors in series:

- Consider three resistors R_1, R_2 and R_3 are connected end to end to form series connection as shown in figure.



(a) Three resistors in series

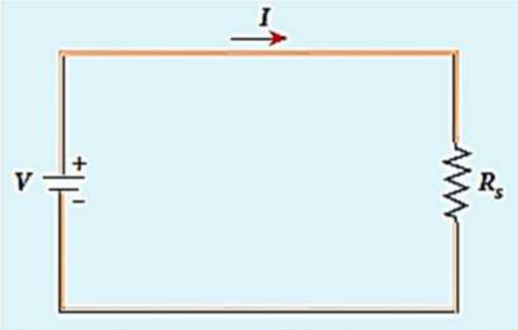
- The amount of charge passing through R_1, R_2 and R_3 are same. But the total voltage V given from the battery is divided in each resistor as,

$$V_1 = IR_1,\ V_2 = IR_2 \text{ and } V_3 = IR_3$$

- Now the total voltage V can written as,

$$V = V_1 + V_2 + V_3$$
$$V = IR_1 + IR_2 + IR_3$$
$$V = I(R_1 + R_2 + R_3)$$
$$V = IR_S$$

Where $R_S = R_1 + R_2 + R_3$ is the equivalent resistance.

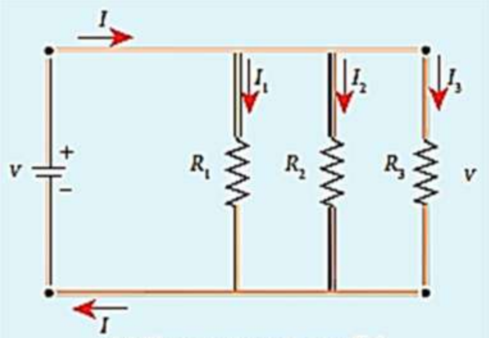


(b) Equivalent resistance (R_s) has the same current

- Thus, when several resistances are connected in series, the total or equivalent resistance is the sum of the individual resistances as shown in the figure above.

(b) Resistors in parallel:

- Consider three resistors R_1, R_2 and R_3 are connected across the same potential to form parallel connection as shown in figure.



(a) Three resistors in parallel

- The total current I leaves from the battery is divided in each resistor as,

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2} \text{ and } I_3 = \frac{V}{R_3}$$

- Now the total current I can written as,

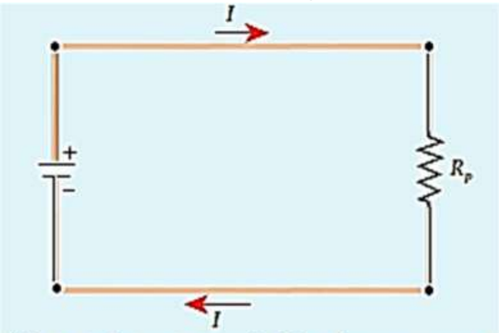
$$I = I_1 + I_2 + I_3$$
$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$
$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$
$$I = \frac{V}{R_P}$$

- Here,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Where R_P is the equivalent resistance of parallel connection.

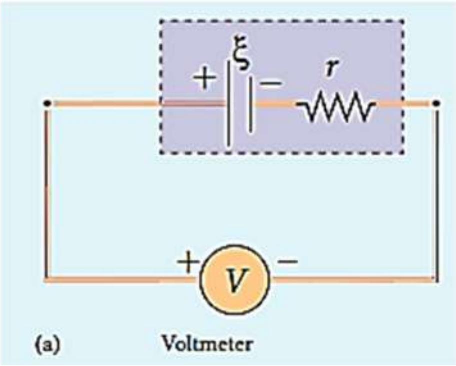
- Thus, when a number of resistors are connected in parallel, the sum of the reciprocal of the values of resistance of the individual resistor is equal to the reciprocal of the effective resistance of the combination as shown in figure below.



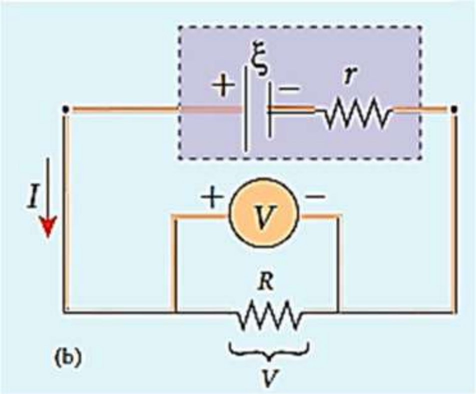
(b) Equivalent resistance (R_p) has the same current

5. Explain the determination of the internal resistance of a cell using voltmeter.

- The emf of cell ξ is measured by connecting a high resistance voltmeter across it without connecting the external resistance R as shown in Figure(a).



- Then, external resistance R is included in the circuit and current I is established in the circuit. The potential difference across R is equal to the potential difference across the cell (V) as shown in Figure(b).



- The potential drop across the resistor R is,

$$V = IR \longrightarrow (1)$$

- Due to internal resistance r of the cell, the voltmeter reads a value V, which is less than the emf of cell ξ. It is because, certain amount of voltage (Ir) has dropped across the internal resistance r.

$$V = \xi - Ir$$

$$Ir = \xi - V \longrightarrow (2)$$

- Dividing equation(2) by(1),we get,

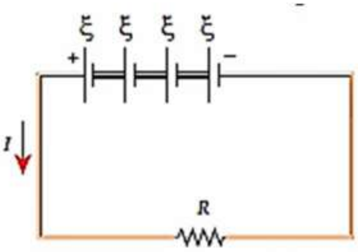
$$\frac{Ir}{IR} = \frac{\xi - V}{V}$$

$$r = \left(\frac{\xi - V}{V}\right) R$$

- Since ξ, V and R are known, internal resistance r can be determined. We can also find the total current that flows in the circuit.

6. Explain about cells in series.

- Suppose n cells, each of emf ξ volts and internal resistance r ohms are connected in series with an external resistance R as shown in Figure.



- The total emf of the battery = nξ
- The total resistance in the circuit = nr + R
- By Ohm's law, the current in the circuit is

$$I = \frac{Total\ emf}{Total\ resistance} = \frac{n\xi}{nr + R}$$

- <u>Case(a) :</u> If r << R, then,

$$I = \frac{n\xi}{R} \approx nI_1$$

Where I_1 is the current due to single cell.

$$I_1 = \frac{\xi}{R}$$

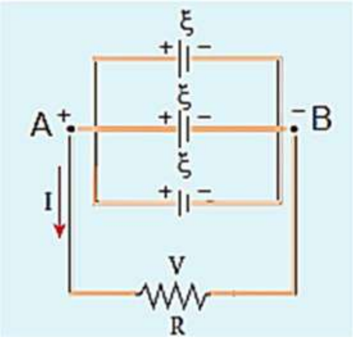
- Thus, if r is negligible when compared to R the current supplied by the battery is n times that supplied by a single cell.
- <u>Case (b) :</u> If r >> R, then,

$$I = \frac{n\xi}{nr} \approx \frac{\xi}{r}$$

- It is the current due to a single cell. That is, current due to the whole battery is the same as that due to a single cell and hence there is no advantage in connecting several cells.
- Thus, series connection of cells is advantageous only when the effective internal resistance of the cells is negligibly small compared with R.

7. Explain about cells in parallel.

- Suppose n cells, each of emf ξ volts and internal resistance r ohms are connected in parallel. An external resistance R is connected between A and B as shown in Figure.



- The equivalent internal resistance of the battery is,

$$\frac{1}{r_{eq}} = \frac{n}{r}$$

$$r_{eq} = \frac{r}{n}$$

- The total resistance = $r_{eq} + R = \frac{r}{n} + R$

- By Ohm's law, the current in the circuit is

$$I = \frac{Total\ emf}{Total\ resistance} = \frac{\xi}{\frac{r}{n} + R}$$

$$I = \frac{n\xi}{r + nR}$$

- **Case(a) :** If r << R, then,

$$I = \frac{\xi}{R}$$

- The above equation implies that current due to the whole battery is the same as that due to a single cell.

- **Case (b) :** If r >> R, then,

$$I = \frac{n\xi}{r} = nI_1$$

Where $I_1 = \frac{\xi}{r}$ is the current due to single cell.

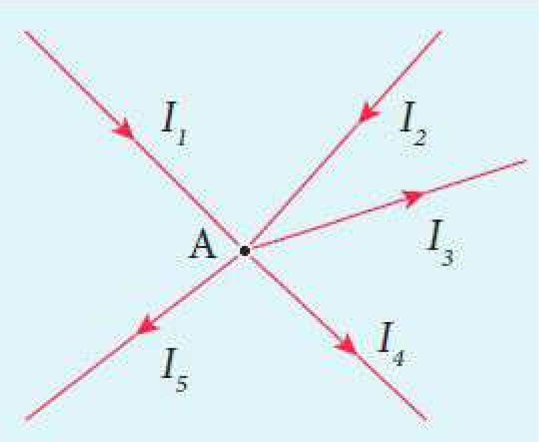
- Thus, the current through the external resistance due to the whole battery is n times the current due to a single cell.

- Hence it is advantageous to connect cells in parallel when the external resistance is very small compared to the internal resistance of the cells.

8. State and explain Kirchhoff's rules.

(i) Kirchhoff's first rule : (Current or Junction rule)

- It states that the algebraic sum of the currents at any junction of a circuit is zero.



- It is a statement of conservation of electric charge. All charges that enter a given junction in a circuit must leave that junction since charge cannot build up or disappear at a junction.

- Current entering the junction is taken as positive and current leaving the junction is taken as negative.

- Applying this law to the junction A in Figure.

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

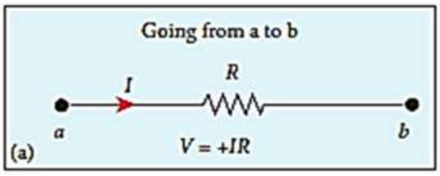
$$I_1 + I_2 = I_3 + I_4 + I_5$$

(i) Kirchhoff's Second rule :(Voltage or Loop rule)

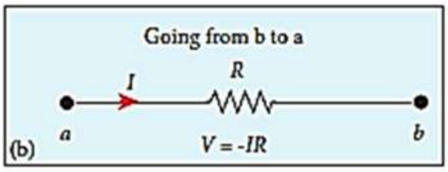
- It states that in a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit.

- This rule follows from the law of conservation of energy for an isolated system (The energy supplied by the emf sources is equal to the sum of the energy delivered to all resistors).

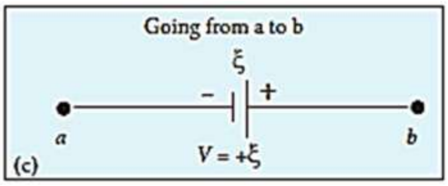
- The product of current and resistance is taken as positive when the direction of the current is followed. (Figure (a))



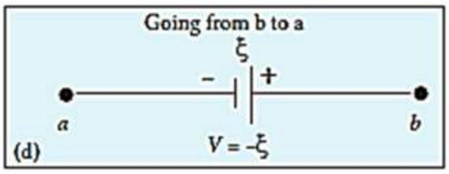
- Suppose if the direction of current is opposite to the direction of the loop, then product of current and voltage across the resistor is negative. (Figure (b))

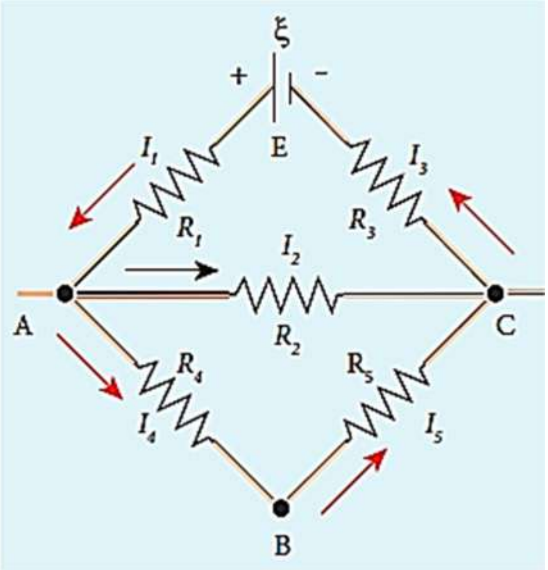


- The emf is considered positive when proceeding from the negative to the positive terminal of the cell (Figure(c)).



- The emf is considered negative when proceeding from the positive to the negative terminal of the cell (Figure(d)).





- Thus, applying Kirchoff's second law to the closed loop EACE,

$$I_1R_1 + I_2R_2 + I_3R_3 = \xi$$

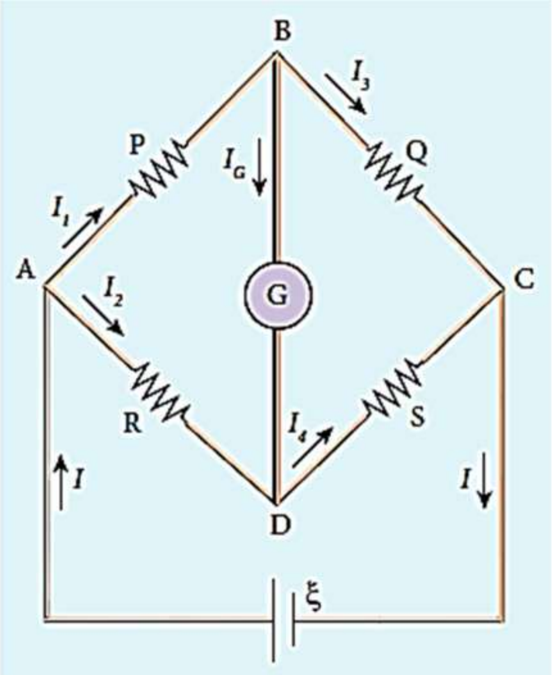
- Applying Kirchoff's second law to the closed loop ABCA,

$$I_4R_4 + I_5R_5 - I_2R_2 = 0$$

9. Obtain the condition for bridge balance in Wheatstone's bridge.

- An important application of Kirchhoff 's rules is the Wheatstone's bridge. It is used to compare resistances and also helps in determining the unknown resistance in electrical network.

- The bridge consists of four resistances P, Q, R and S are connected as shown in Figure.



- A galvanometer G is connected between the points B and D. The battery is connected between the points A and C. The current through the galvanometer is I_G and its resistance is G.

- Applying Kirchhoff 's current rule to junction B

$$I_1 - I_G - I_3 = 0 \longrightarrow (1)$$

- Applying Kirchhoff 's current rule to junction D

$$I_2 + I_G - I_4 = 0 \longrightarrow (2)$$

- Applying Kirchhoff 's voltage rule to loop ABDA,

$$I_1P + I_GG - I_2R = 0 \longrightarrow (3)$$

- Applying Kirchhoff 's voltage rule to loop ABCDA,

$$I_1P + I_3Q - I_4S - I_2R = 0 \longrightarrow (4)$$

- When the points B and D are at the same potential, the bridge is said to be balanced. In this condition $I_G = 0$. Substituting this in equations(1),(2) &(3), we get,

$$I_1 = I_3 \longrightarrow (5)$$
$$I_2 = I_4 \longrightarrow (6)$$
$$I_1P = I_2R \longrightarrow (7)$$

- Substituting the equations(5) &(6) in equation(4), we get,

$$I_1P + I_1Q - I_2S - I_2R = 0$$

$$I_1(P+Q) = I_2(R+S) \longrightarrow (8)$$

- Dividing equation(8) by equation(7), we have,

$$\frac{P+Q}{P} = \frac{R+S}{R}$$
$$1 + \frac{Q}{P} = 1 + \frac{S}{R}$$
$$\frac{Q}{P} = \frac{S}{R}$$
$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

- This is the bridge balance condition. If three of the resistances are known, the value of unknown resistance (fourth one) can be determined.

10. Explain the determination of unknown resistance using meter bridge.

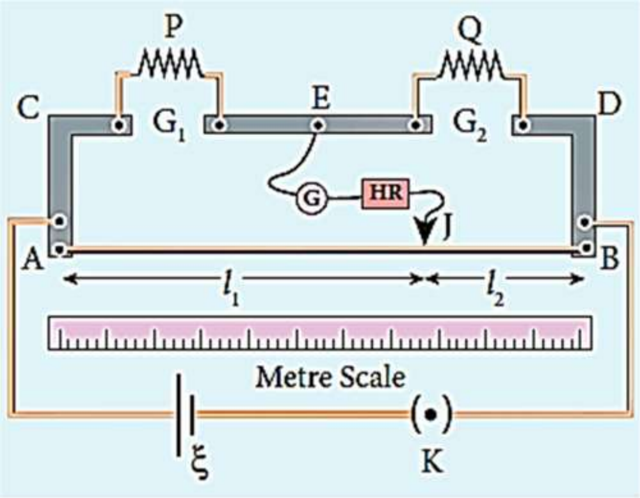
- The meter bridge is another form of Wheatstone's bridge. It consists of a uniform manganin wire AB of one meter length.

- This wire is stretched along a meter scale on a wooden board between two copper strips C and D.

- Between these two copper strips another copper strip E is mounted to enclose two gaps G_1 and G_2 as shown in Figure.

- An unknown resistance P is connected in G_1 and a standard resistance Q is connected in G_2.

- A jockey (conducting wire) is connected to the terminal E on the central copper strip through a galvanometer (G) and a high resistance (HR).
- The exact position of jockey on the wire can be read on the scale. A Lechlanche cell and a key (K) are connected across the ends of the bridge wire.



- The position of the jockey on the wire is adjusted so that the galvanometer shows zero deflection. Let the point be J.
- The lengths AJ and JB of the bridge wire now replace the resistance R and S of the Wheatstone's bridge. Then,

$$\frac{P}{Q}=\frac{R}{S}=\frac{R'.AJ}{R'.JB}$$

where R′ is the resistance per unit length of wire.

$$\frac{P}{Q}=\frac{AJ}{JB}=\frac{l_1}{l_2}$$

$$P=Q\frac{l_1}{l_2}$$

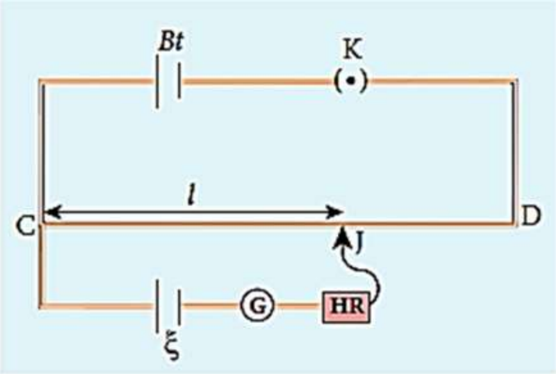
- The end resistance is created at the ends of the copper strips due to soldering. To eliminate this, another set of readings are taken with P and Q interchaged. From this the average value of P is found.
- To find the specific resistance of the material of the wire in the coil P, the radius r and length l of the wire are measured.
- The specific resistance or resistivity ρ can be calculated using the relation,

$$\rho=P\times\frac{A}{l}$$

$$\rho=P\times\frac{\pi r^2}{l} \qquad [\because A=\pi r^2]$$

⑪ Explain the potentiometer and its principle.

- Potentiometer is used for the accurate measurement of potential differences, current and resistances.
- It consists of 10 m long uniform wire of manganin or constantan stretched in parallel rows each of 1 meter length, on a wooden board.
- The two free ends A and B are brought to the same side and fixed to copper strips with binding screws.
- A meter scale is fixed parallel to the wire.
- The principle of the potentiometer is illustrated in Figure.



- A steady current is maintained across the wire CD by a battery Bt. The battery, key and the potentiometer wire are connected in series forms the primary circuit.
- The positive terminal of a primary cell of emf ξ is connected to the point C and negative terminal is connected to the jockey through a galvanometer G and a high resistance HR. This forms the secondary circuit.
- Let contact be made at any point J on the wire by jockey where galvanometer shows zero. Now CJ is the balancing length l. In this position,

$$\begin{pmatrix}\text{The emf of}\\ \text{the cell}\end{pmatrix}=\begin{pmatrix}\text{potential difference}\\ \text{across CJ}\end{pmatrix}$$

$$\xi = Irl$$

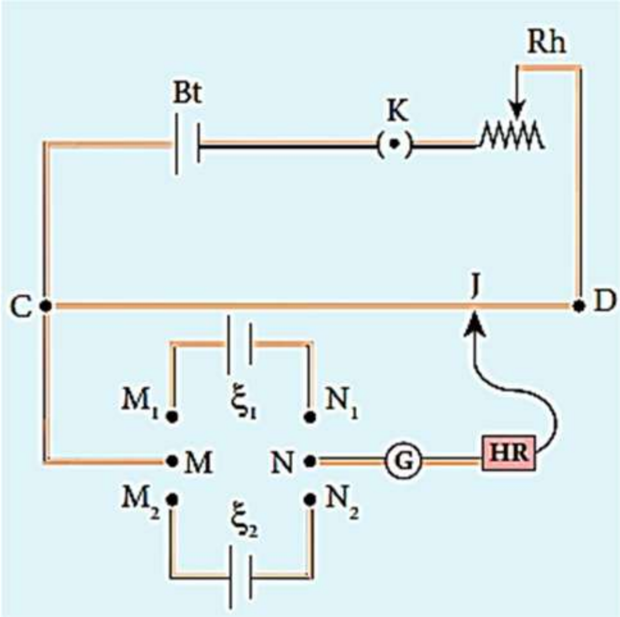
- Since l and r are constants,

$$\xi \propto l$$

- The emf of the cell is directly proportional to the balancing length.

⑫ How the emf of two cells are compared using potentiometer?

❖ To compare the emf of two cells, the circuit connections are made as shown in Figure.



❖ Potentiometer wire CD is connected to a battery Bt and a key K in series. This is the primary circuit.

❖ The end C of the wire is connected to the terminal M of a DPDT (Double Pole Double Throw) switch and the other terminal N is connected to a jockey through a galvanometer G and a high resistance HR.

❖ The cells whose emf ξ_1 and ξ_2 to be compared are connected to the terminals M_1,N_1 and M_2,N_2 of the DPDT switch. The positive terminals of Bt, ξ_1 and ξ_2 should be connected to the same end C.

❖ The DPDT switch is pressed towards M_1, N_1 so that cell ξ_1 is included in the secondary circuit and the balancing length l_1 is found by adjusting the jockey for zero deflection.

❖ Then the second cell ξ_2 is included in the circuit and the balancing length l_2 is determined.

❖ Let r be the resistance per unit length of the potentiometer wire and I be the current flowing through the wire. we have,

$$\xi_1 = Irl_1 \longrightarrow (1)$$

$$\xi_2 = Irl_2 \longrightarrow (2)$$

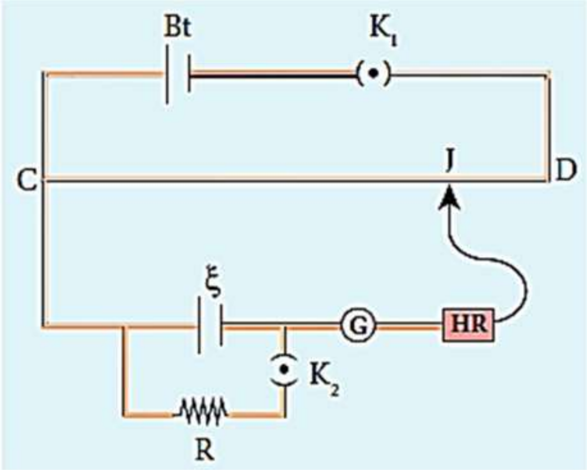
❖ Dividing equation (1) by (2), we get,

$$\frac{\xi_1}{\xi_2} = \frac{l_1}{l_2}$$

❖ By including a rheostat (Rh) in the primary circuit, the experiment can be repeated several times by changing the current flowing through it.

13. Explain the determination of the internal resistance of a cell using potentiometer.

❖ To measure the internal resistance of a cell, the circuit connections are made as shown in Figure.



❖ The end C of the potentiometer wire is connected to the positive terminal of the battery Bt and the negative terminal of the battery is connected to the end D through a key K_1. This forms the primary circuit.

❖ The positive terminal of the cell ξ whose internal resistance is to be determined is also connected to the end C of the wire.

❖ The negative terminal of the cell ξ is connected to a jockey through a galvanometer and a high resistance.

❖ A resistance box R and key K_2 are connected across the cell ξ. With K_2 open, the balancing point J is obtained and the balancing length CJ = l_1 is measured. Since the cell is in open circuit, its emf is

$$\xi \propto l_1 \longrightarrow (1)$$

❖ A suitable resistance (say, 10 Ω) is included in the resistance box and key K_2 is closed. Let r be the internal resistance of the cell.

❖ The current passing through the cell and the resistance R is given by,

$$I = \frac{\xi}{R + r}$$

❖ The potential difference across R is,

$$V = \frac{\xi R}{R + r} \qquad [\because V = IR]$$

❖ When this potential difference is balanced on the potentiometer wire at the balancing length l_2, we can write,

$$\frac{\xi R}{R + r} \propto l_2 \longrightarrow (2)$$

- Dividing equation(1) by(2), we get,

$$\frac{R+r}{R}=\frac{l_1}{l_2}$$

$$1+\frac{r}{R}=\frac{l_1}{l_2}$$

$$r=R\left[\frac{l_1}{l_2}-1\right]$$

$$r=R\left[\frac{l_1-l_2}{l_2}\right]$$

- Substituting the values of the R, l_1 and l_2, the internal resistance of the cell is determined.

- It is found that the internal resistance of the cell is proportional to the external resistance.

14. Elucidate the applications of Joule's heating effect.

(a) Electric heaters :

- Electric iron, electric heater, electric toaster shown in figure are some of the home appliances that utilize the heating effect of current.


Electric Iron box

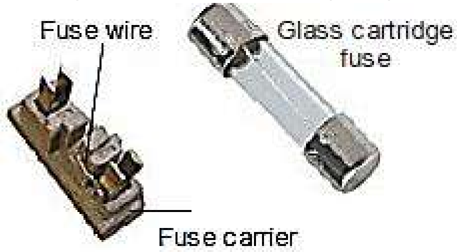

Electric water heater


Electric Toaster

- In these appliances, the heating elements are made of nichrome, an alloy of nickel and chromium. Nichrome has a high specific resistance and can be heated to very high temperatures without oxidation.

(b) Electric fuses :

- Fuses are short length of wire made up of low melting material like copper(35 A) or lead(5 A).



- It protects the electric devices from the passage excess current, which may damage them.

- Fuse melts and breaks the circuit if current exceeds a certain value.

- The only disadvantage of it is that once fuse wire is burnt due to excessive current, they need to be replaced.

- Now-a-days in houses, circuit breakers (trippers) are also used instead of fuses.


MCB - Miniature Circuit Breaker

- Whenever there is an excessive current produced due to faulty wire connection, the circuit breaker switch opens. After repairing the faulty connection, we can close the circuit breaker switch.

(c) Electric furnace :

- Furnaces are used to manufacture a large number of technologically important materials such as steel, silicon carbide, quartz, gallium arsenide, etc).

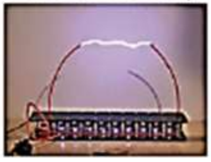
- To produce temperatures up to 1500^{0}C, molybdenum-nichrome wire wound on a silica tube is used. Carbon arc furnaces produce temperatures up to 3000^{0}C.

(c) Electric lamp :

- It consists of a tungsten filament (melting point 3380^{0}C) kept inside a glass bulb and heated to incandescence by current.

Electric discharge lamp

Electric arc lamp



Incandescent electric lamp

Electric welding

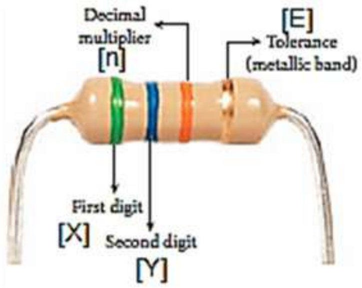
- In incandescent electric lamps only about 5% of electrical energy is converted into light and the rest is wasted as heat.

- Electric discharge lamps, electric welding and electric arc also utilize the heating effect of current

Note :

Finding the value of resistance by using colour codes in carbon resistors :

Carbon resistors consists of a ceramic core, on which a thin layer of crystalline carbon is deposited.



Formula : R = [X] [Y] x $10^n \pm E$

Here **X, Y, n** are colour indices denote resistance value and **E** is the tolerence. R is the resistance value of carbon resistor.

Colour	Tolerence (E)
Gold	5 %
Silver	10 %
Colourless	20 %

Colour	Index		
	n	X	Y
Black	0	0	0
Brown	1	1	1
Red	2	2	2
Orange	3	3	3
Yellow	4	4	4
Green	5	5	5
Blue	6	6	6
Violet	7	7	7
Grey	8	8	8
White	9	9	9

Memento:

The above colour coding system can be reminded by the acronym as **BBROY Great Britain Very Good Wife.**

(E.g) Determination of resistance value of carbon resistor consists of Orange - Orange – Orange colour rings :

(X) (Y) (n)

X=3 ; Y= 3 ; n = 3

∴Value of resistance **R =** $33 \times 10^3\ \Omega$ (A) 33KΩ

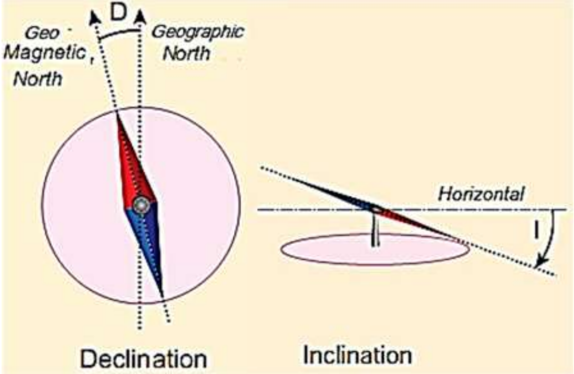
3. MAGNETISM AND MAGNETIC EFFECTS OF ELECTRIC CURRENT

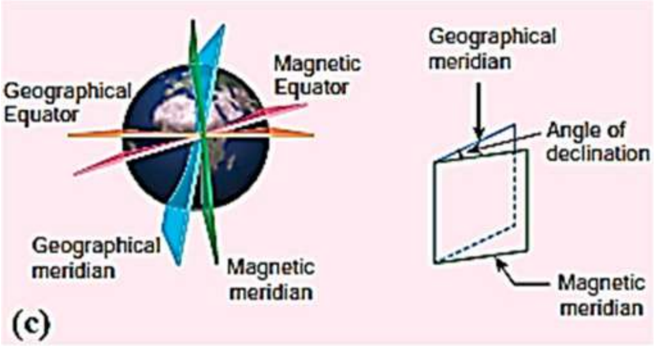
1. What is Geomagnetism or terrestrial magnetism?

The branch of physics which deals with the Earth's magnetic field is called Geomagnetism or Terrestrial magnetism.

2. What are the elements of earth's magnetic field?

- Magnetic declination (D)
- Magnetic dip or inclination (I)
- The horizontal component of the Earth's magnetic field (B_H)





- **Geographic axis** – An axis about which Earth spins.
- **Geographic meridian** – A vertical plane passing through the geographic axis.
- **Geographic equator** - A great circle perpendicular to geographic axis.
- **Magnetic axis** - The straight line that connects magnetic poles of Earth.
- **Magnetic meridian** - A vertical plane passing through magnetic axis.
- **Magnetic equator** - A great circle perpendicular to Earth's magnetic axis.

3. What is declination or magnetic declination?

The angle between magnetic meridian at a point and geographical meridian is called the declination or magnetic declination (D). [In chennai, $D = -1^0 8'$(west)]

4. What is magnetic inclination?

The angle subtended by the Earth's total magnetic field $\vec{B}$ with the horizontal direction in the magnetic meridian is called dip or magnetic inclination(I) at that point. [For chennai, $I = 14^0 16'$]

5. What is horizontal component of Earth's magnetic field?

The component of Earth's magnetic field along the horizontal direction in the magnetic meridian is called horizontal component of Earth's magnetic field (B_H).

6. Define magnetic dipole moment. Give its unit.

The magnetic dipole moment($\vec{p}_m$) is defined as the product of its pole strength and magnetic length. It is a vector quantity. Its unit is A m.

$$\vec{p}_m = q_m \vec{d} \qquad (or) \qquad p_m = 2q_m l$$

7. Define magnetic field. Give its unit.

The magnetic field $\vec{B}$ at a point is defined as a force experienced by the bar magnet of unit pole strength. Its unit is N A^{-1} m^{-1} .

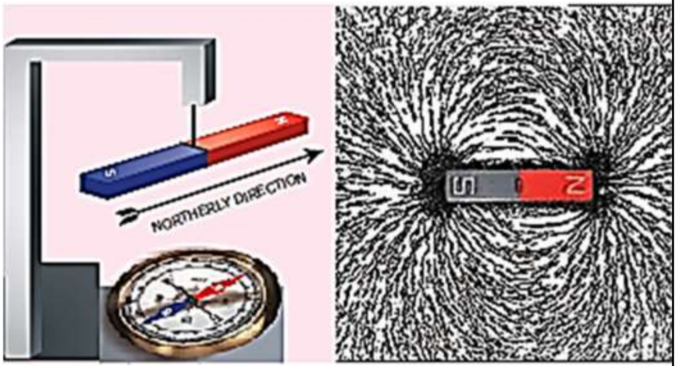
$$\vec{B} = \frac{\vec{F}}{q_m}$$

8. What are the types magnets?

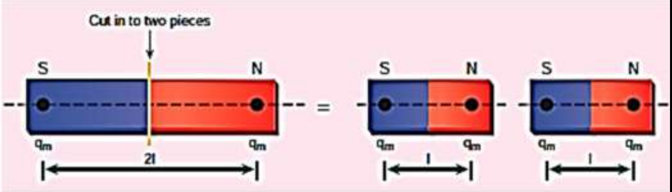
- Natural magnets.
  __Ex:__ Iron, Cobalt, nickel, etc.,
- Artificial magnets.
  __Ex:__ Bar magnet (Rectangle or cylinder shape).

9. What are the properties of magnet?

- A freely suspended bar magnet will always point along the north-south direction.

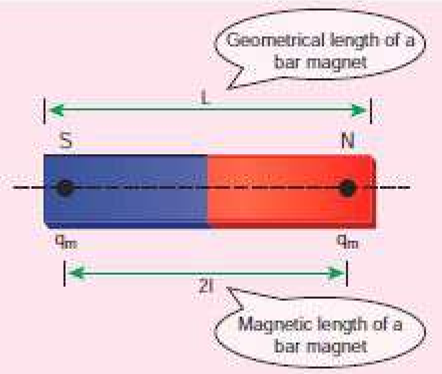


- A magnet attracts another magnet or magnetic substances towards itself. The attractive force is maximum near the end of the bar magnet. When a bar magnet is dipped into iron filling, they cling to the ends of the magnet.
- When a magnet is broken into pieces, each piece behaves like a magnet with poles at its ends.



- Two poles of a magnet have pole strength equal to one another.

- The length of the bar magnet is called geometrical length and the length between two magnetic poles in a bar magnet is called magnetic length. Magnetic length is always slightly smaller than geometrical length. The ratio of magnetic length and geometrical length is $\frac{5}{6} = 0.833$.



10. What are the properties of magnetic field lines?

- Magnetic field lines are continuous closed curves. The direction of magnetic field lines is from North pole to South pole outside the magnet and South pole to North pole inside the magnet.

- The direction of magnetic field at any point on the curve is known by drawing tangent to the magnetic line of force at that point.

- Magnetic field lines never intersect each other. Otherwise, the magnetic compass needle would point towards two directions, which is not possible.

- The degree of closeness of the field lines determines the relative strength of the magnetic field. The magnetic field is strong where magnetic field lines crowd and weak where magnetic field lines thin out.

11. What is magnetic flux? Give its unit.

The number of magnetic field lines crossing normal to the given area is called magnetic flux Φ_B. Its unit is weber(Wb). [1 weber = 10^8 maxwell]

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

12. Define magnetic flux density. Give its unit.

The magnetic flux density $\vec{B}$ is defined as the number of magnetic field lines crossing unit area kept normal to the direction of line of force. Its unit is Wb m^{-2} or tesla.

13. What is uniform magnetic field? Give an example.

The magnetic field which has same magnitude and direction at all the points in a given region is called uniform magnetic field. <u>Ex:</u> Local Earth's magnetic field.

14. What is non-uniform magnetic field? Give an example.

The magnetic field whose magnitude or direction or both varies at all its points is called non-uniform magnetic field. Ex: Magnetic field of a bar magnet.

15. State Coulomb's inverse square law of magnetism.

The force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.

$$\vec{F} = k\frac{q_{m_A} q_{m_B}}{r^2}\hat{r} = \frac{\mu_0}{4\pi}\frac{q_{m_A} q_{m_B}}{r^2}\hat{r}$$

16. State tangent law.

When a magnetic needle or magnet is freely suspended in two mutually perpendicular uniform magnetic fields, it will come to rest in the direction of the resultant of the two fields.

17. What is magnetising field? Give its unit.

The magnetic field which is used to magnetize a sample or specimen is called the magnetising field$(\vec{H})$. Magnetising field is a vector quantity. Its unit is A m^{-1}.

18. Define magnetic permeability.

The magnetic permeability μ can be defined as the measure of ability of the material to allow the passage of magnetic field lines through it.

19. Define relative permeability.

The relative permeability μ_r is defined as the ratio between absolute permeability(μ) of the medium to the permeability of free space(μ_0). For free space(air or vacuum), $\mu_r = 1$.

$$\mu_r = \frac{\mu}{\mu_0}$$

20. What is intensity of magnetisation? Give its unit.

The net magnetic moment per unit volume of the material is known as intensity of magnetisation or magnetisation vector or magnetisation. Its unit is A m^{-1}.

$$\vec{M} = \frac{\vec{p}_m}{V} = \frac{q_m}{A}$$

21. What is magnetic susceptibility?

Magnetic susceptibility is defined as the ratio of the intensity of magnetisation $(\vec{M})$ induced in the material due to the magnetising field to the magnetising field $(\vec{H})$.

$$\chi_m = \frac{|\vec{M}|}{|\vec{H}|}$$

22. What is magnetic induction or total magnetic field?

The magnetic induction (total magnetic field) inside the specimen $\vec{B}$ is equal to the sum of the magnetic field $\vec{B}_0$ produced in vacuum due to the magnetising field and the magnetic field $\vec{B}_m$ due to the induced magnetisation of the substance.

$$\vec{B} = \vec{B}_0 + \vec{B}_m = \mu_0(\vec{H} + \vec{I})$$

23. What are the types magnetic materials?

- Diamagnetic materials.
- Paramagnetic materials.
- Ferromagnetic materials.

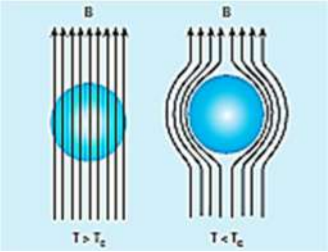
24. What are diamagnetic materials? Give examples.

The magnetic materials, which when placed at external magnetic field get oppositely magnetised to that field are called diamagnetic materials.

Ex: Bismath, Copper and Water.

25. What is Meissner effect?

The expulsion of magnetic flux from a superconductor during its transition to the superconducting state is known as Meissner effect.



26. What are paramagnetic materials? Give examples.

The magnetic materials, which when placed at external magnetic field get weakly magnetised along that field are called paramagnetic materials.

Ex: Aluminium, Platinum and Chromium.

27. What are ferromagnetic materials? Give examples.

The magnetic materials, which when placed at external magnetic field get strongly magnetised along that field are called diamagnetic materials.

Ex: Iron, Nickel and Cobalt.

28. Compare the properties of dia, para and ferro magnetic materials.

S. No.	Diamagnetic material	Paramagnetic material	Ferromagnetic material
1.	Magnetic susceptibility is negative.	Magnetic susceptibility is positive and small.	Magnetic susceptibility is positive and large.
2.	Relative permeability is slightly less than unity.	Relative permeability is greater than unity.	Relative permeability is large.
3.	The magnetic field lines are repelled or expelled when placed in a magnetic field.	The magnetic field lines are attracted when placed in a magnetic field.	The magnetic field lines are strongly attracted when placed in a magnetic field.
4.	Susceptibility is nearly temperature independent.	Susceptibility is inversely proportional to temperature.	Susceptibility is inversely proportional to temperature.
5.	In a non-uniform magnetic field, it tends to move from stronger to weaker part of the filed.	In a non-uniform magnetic field, it tends to move from weaker to stronger part of the filed.	In a non-uniform magnetic field, it strongly tends to move from weaker to stronger part of the filed.

29. State Curie's law.

The magnetic susceptibility of the paramagnetic substance is inversely proportional to the absolute temperature.

$$\chi_m \propto \frac{1}{T} \quad or \quad \chi_m = \frac{C}{T} \quad ; C - Curie\ constant$$

30. State Curie-Weiss law.

The magnetic susceptibility of the ferromagnetic substance is inversely proportional to the difference of absolute and Curie temperature.

$$\chi_m \propto \frac{1}{T - T_C} \quad or \quad \chi_m = \frac{C}{T - T_C} \quad ; C - Curie\ constant$$

31. What is meant by Curie temperature?

The temperature at which ferromagnetic material becomes paramagnetic is called Curie temperature T_C.

32. Define remanence or retentivity.

Remanence or Retentivity is defined as the ability of the materials to retain the magnetism in them even magnetising field vanishes.

33. What is coercivity?

The magnitude of the reverse magnetising field for which the residual magnetism of the material vanishes is called its coercivity.

34. What is hysteresis?

The phenomenon of lagging of magnetic induction behind the magnetising field is called hysteresis.

35. What is hysteresis loop?

The graph between magnetic induction and the magnetising field is called hysteresis loop.

36. What is hysteresis loss?

The energy loss during one cycle of magnetisation of the ferromagnetic substance is called hysteresis loss.

37. What are the applications of hysteresis loop?

- Hysteresis loop provides information such as retentivity, coercivity, permeability, susceptibility and energy loss during one cycle of magnetisation for each ferromagnetic material.
- It is used to selecting proper and suitable material for a given purpose. Ex: Permanent magnets, Electromagnets and core of transformer.

38. What kind of properties the materials should have for making permanent magnets? Give the examples.

The materials with high retentivity, high coercivity and high permeability are suitable for making permanent magnets.

Ex: Steel and Alnico(Al+Ni+C_o).

39. What kind of properties the materials should have for making electromagnets? Give the examples.

The materials with high initial permeability, low retentivity, low coercivity and thin hysteresis loop with smaller area are preferred to make electromagnets.

Ex: Soft iron and Mumetal (Nickel Iron alloy).

40. What kind of properties the materials should have for making core of transformer? Give an example.

The materials with high initial permeability, large magnetic induction and thin hysteresis loop with smaller area are needed to design transformer cores.

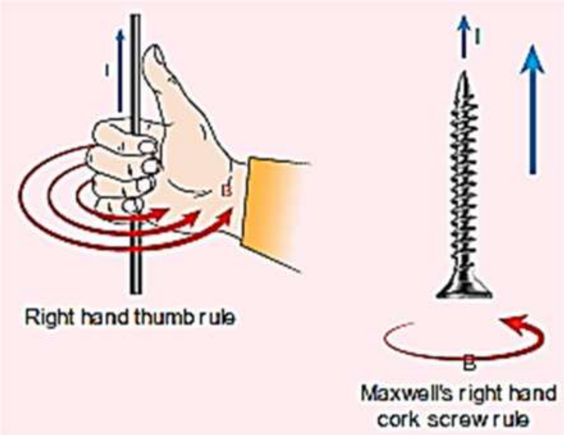
Ex: Soft iron.

41. What are the difference between soft and hard ferromagnetic materials?

S. No.	Properties	Soft ferromagnetic material	Hard ferromagnetic material
1.	When external field is removed	Magnetisation disappears.	Magnetisation persists.
2.	Area of the loop	Small	Large
3.	Retentivity	Low	High
4.	Coercivity	Low	High
5.	Susceptibility and magnetic permeability	High	Low
6.	Hysteresis loss	Less	More
7.	Uses	Solenoid core, transformer core and electromagnets.	Permanent magnets.
8.	Examples	Soft iron, Mumetal, Stalloy etc.	Steel, Alnico, Lodestone etc.

42. State right hand thumb rule for magnetic field.

If we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flow, then the fingers encircling the wire points in the direction of the magnetic field lines produced.



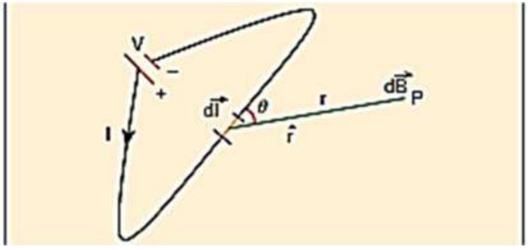
43. State Maxwell's right hand cork screw rule.

When a right-handed screw is rotated using a screw driver, the screw advances along the direction of current and the direction of rotation of the screw points the direction of the magnetic field.

44. State Biot-Savart law.

The magnitude of magnetic field $\overrightarrow{dB}$ at a point P at a distance r from the small elemental length taken on a conductor carrying current varies

- directly as the strength of the current I
- directly as the magnitude of the length element $\overrightarrow{dl}$
- directly as the sine of the angle (say,θ) between $\overrightarrow{dl}$ and $\hat{r}$.
- inversely as the square of the distance between the point P and length element $\overrightarrow{dl}$.



$$dB = \frac{\mu_0}{4\pi}\frac{I\, dl\, sin\theta}{r^2}$$

45. What are the similarities between Coulomb's law and Biot-Savart's law?

S. No.	Coulomb' Law	Biot-Savart's Law
1.	Electric field obeys inverse square law.	Magnetic field obeys inverse square law.
2.	Electric field is long range field.	Magnetic field is also long range field.
3.	Electric field obeys the principle of superposition	Magnetic field also obeys the principle of superposition.
4.	Electric field is linear with respect to source. $i.e.\ \ E \propto q$	Magnetic field is linear with respect to source. $i.e.\ \ B \propto Idl$

46. Distinguish between Coulomb's law and Biot-Savart's law.

S. No.	Coulomb' Law	Biot-Savart's Law
1.	$\vec{E}$ is Produced by a scalar source i.e., an electric charge q.	$\vec{B}$ Produced by a vector source i.e., current element $I\,\overrightarrow{dl}$
2.	$\vec{E}$ is directed along the position vector joining the source and the point at which the field is calculated.	$\vec{B}$ is directed perpendicular to the position vector $\vec{r}$ and the current element $I\,\overrightarrow{dl}$
3.	$\vec{E}$ does not depend on angle.	$\vec{B}$ depends on the angle between the position vector $\vec{r}$ and the current element $I\,\overrightarrow{dl}$

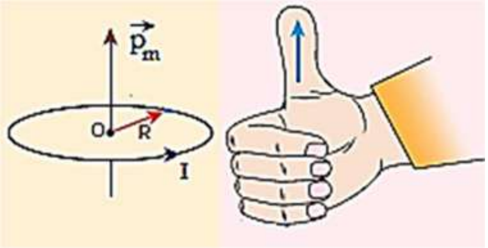
47. What is magnetic dipole moment of a current loop?

The magnetic dipole moment of any current loop is equal to the product of the current and area of the loop.

$$\vec{p}_m = I\vec{A}$$

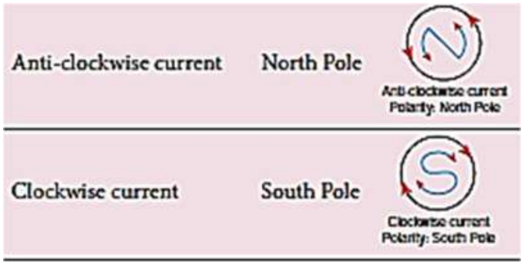
48. State right hand thumb rule for magnetic moment.

If we curl the fingers of right hand in the direction of current in the loop, then the stretched thumb gives the direction of the magnetic moment associated with the loop.



49. State end rule for polarity of current loop.

If the current in circular loop is anti-clockwise direction, current loop acts as north pole whereas if the current is clockwise direction, current loop acts as south pole.



Anti-clockwise current	North Pole	Anti-clockwise current Polarity: North Pole
Clockwise current	South Pole	Clockwise current Polarity: South Pole

50. Define gyro-magnetic ratio. Give its value.

The gyro-magnetic ratio is defined as the ratio between magnetic dipole moment of the electron and angular momentum of the electron.

$$\left.\begin{matrix} Gyro-magnetic \\ ratio \end{matrix}\right\} = \frac{\mu_L}{L} = \frac{e}{2m} = 8.78 \times 10^{10} C\ kg^{-1}$$

51. What is Bohr magneton? Give its value.

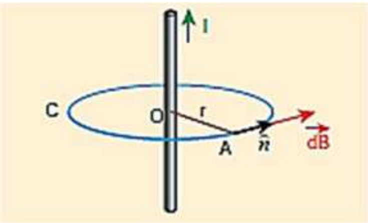
The minimum magnetic moment of an electron is called Bohr magneton.

$$\mu_B = (\mu_L)_{min} = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} A\ m^2$$

52. State Ampere's circuital law.

The line integral of magnetic field over a closed loop is μ_0 times net current enclosed by the loop.

$$\oint_C \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{encl}$$

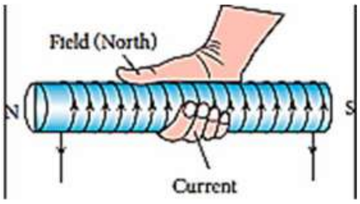


53. What is solenoid?

A solenoid is a long coil of wire closely wound in the form of helix.

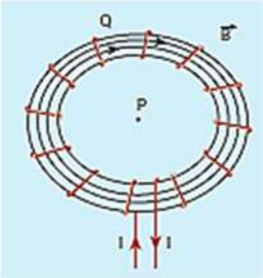
54. State right hand palm rule for solenoid.

The current carrying solenoid is held in right hand. If the fingers curl in the direction of current, then extended thumb gives the direction of magnetic field of current carrying solenoid.



55. What is toroid?

A solenoid is bent in such a way its ends are joined together to form a closed ring shape, is called a toroid.



56. What are the characteristics of Lorentz force?

- $\vec{F}_m$ is directly proportional to the magnetic field $\vec{B}$.
- $\vec{F}_m$ is directly proportional to the velocity $\vec{v}$.
- $\vec{F}_m$ is directly proportional to sine of the angle between the velocity and magnetic field.
- $\vec{F}_m$ is directly proportional to the magnitude of the charge q.
- The direction of $\vec{F}_m$ is always perpendicular to $\vec{v}$ and $\vec{B}$ as $\vec{F}_m$ is the cross product of $\vec{v}$ and $\vec{B}$.
- The direction of $\vec{F}_m$ on negative charge is opposite to the direction of $\vec{F}_m$ on positive charge provided other factors are identical.
- If velocity $\vec{v}$ of the charge q is along magnetic field $\vec{B}$ then, $\vec{F}_m$ is zero.

$$\vec{F}_m = q(\vec{v} \times \vec{B}) \text{ (or) } F_m = Bqv\ sin\theta$$

57. Define 1 tesla.

The strength of the magnetic field is one tesla if unit charge moving in it with unit velocity experiences unit force.

58. State the principle of Cyclotron.

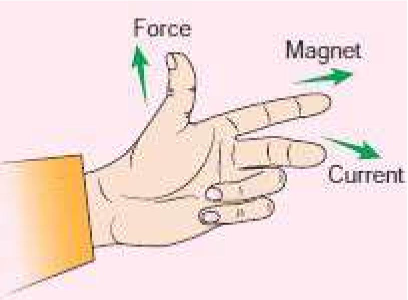
When a charged particle moves normal to the magnetic field, it experiences magnetic Lorentz force.

(59) What are limitations of Cyclotron?

- ❖ The speed of the ion is limited.
- ❖ Electron cannot be accelerated.
- ❖ Uncharged particles cannot be accelerated.

(60) State Fleming's left hand rule.

Stretch forefinger, the middle finger and the thumb of the left hand such that they are in mutually perpendicular directions. If we keep the forefinger in the direction of magnetic field, the middle finger in the direction of the electric current, then the thumb points in the direction the force experienced by the conductors.



(61) Define 1 Ampere.

One ampere is defined as that current when it is passed through each of the two infinitely long parallel straight conductors kept at a distance of one meter apart in vacuum causes each conductor to experience a force of 2 X 10^{-7} newton per meter length of conductor.

62. What is moving coil galavanometer?

Moving coil galvanometer is an instrument used for the detection and measurement of small currents.

63. State the principle of moving coil galvanometer.

When a current carrying loop is placed in a uniform magnetic field it experiences a torque.

64. Define Figure of merit of a galvanometer.

Figure of merit of a galvanometer is defined as the current which produces a deflection of one scale division in the galvanometer.

65. Define Current sensitivity of a galvanometer.

Current sensitivity of a galvanometer is defined as the deflection produced per unit current flowing through it.

$$I_S = \frac{\theta}{I} = \frac{NAB}{K} = \frac{1}{G}$$

(66) How can increase the current sensitivity of a galvanometer?

- ❖ Increasing the number of turns (N) .
- ❖ Increasing the magnetic induction (B) .
- ❖ Increasing the area of the coil (A).
- ❖ Decreasing the couple per unit twist of the suspension wire (K).

67. Why is phosphor-bronze wire used as the suspension wire in the galvanometer?

Phosphor - bronze wire is used as the suspension wire in the galvanometer because the couple per unit twist is very small.

68. Define Voltage sensitivity of a galvanometer.

Voltage sensitivity of a galvanometer is defined as the deflection produced per unit voltage applied across it.

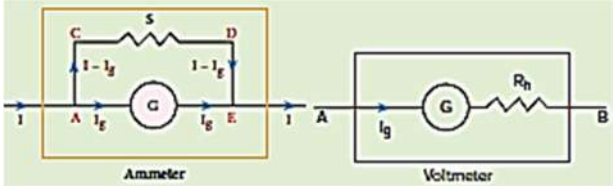
$$V_S = \frac{\theta}{IR_g} = \frac{NAB}{KR_g} = \frac{1}{GR_g} = \frac{I_S}{R_g}$$

69. What is an Ammeter?

Ammeter is an instrument used to measure current in an electrical circuit.

70. How can convert a galvanometer to an Ammeter?

A galvanometer is converted into an ammeter by connecting a low resistance in parallel with the galvanometer.



71. What is a Voltmeter?

Voltmeter is an instrument used to measure potential difference across any element in an electrical circuit.

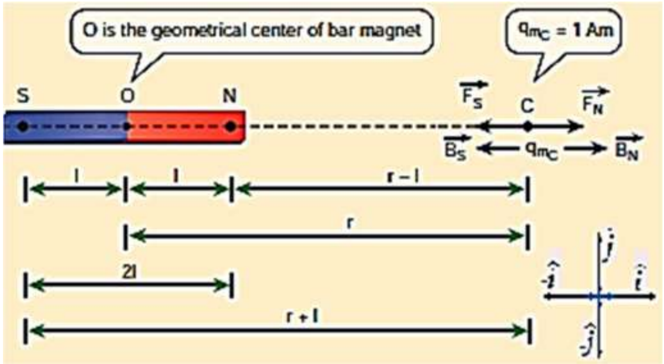
(72) How can convert a galvanometer to a voltmeter?

A galvanometer is converted into a voltmeter by connecting a high resistance in series with the galvanometer.

5 Marks Q & A:

1. Calculate the magnetic induction at a point on the axial line of a bar magnet.

- Consider a bar magnet NS as shown in Figure. Let N be the North Pole and S be the south pole of the bar magnet, each of pole strength qq_m and separated by a distance of 2*l*.
- The magnetic field at a point C lies along the axis of the magnet at a distance r from the geometrical center O of the bar magnet can be computed by keeping unit north pole (q_{mc} = 1 A m) at C.



- The magnetic induction (force experienced by the unit north pole) at C due to north pole,

$$\vec{B}_N = \frac{\vec{F}_N}{q_{m_c}} = \frac{\mu_0}{4\pi}\frac{q_m}{(r-l)^2}\hat{\imath}$$

- The magnetic induction (force experienced by the unit north pole) at C due to south pole,

$$\vec{B}_S = \frac{\vec{F}_S}{q_{m_c}} = \frac{\mu_0}{4\pi}\frac{q_m}{(r+l)^2}(-\hat{\imath})$$

- The net magnetic induction,

$$\vec{B} = \vec{B}_N + \vec{B}_S$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q_m}{(r-l)^2}\hat{\imath} - \frac{\mu_0}{4\pi}\frac{q_m}{(r+l)^2}\hat{\imath}$$

$$\vec{B} = \frac{\mu_0 q_m}{4\pi}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]\hat{\imath}$$

$$\vec{B} = \frac{\mu_0 q_m}{4\pi}\left[\frac{(r+l)^2 - (r-l)^2}{[(r-l)(r+l)]^2}\right]\hat{\imath}$$

$$\vec{B} = \frac{\mu_0 q_m}{4\pi}\left[\frac{\cancel{r^2} + \cancel{l^2} + 2rl - \cancel{r^2} - \cancel{l^2} + 2rl}{(r^2-l^2)^2}\right]\hat{\imath}$$

$$\vec{B} = \frac{\mu_0 q_m}{4\pi}\left[\frac{4rl}{(r^2-l^2)^2}\right]\hat{\imath}$$

- Since r >> *l* , $(r^2 - l^2)^2 \approx r^4$

$$\vec{B} = \frac{\mu_0 q_m}{4\pi}\left[\frac{4rl}{r^4}\right]\hat{\imath}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{2(q_m 2l)}{r^3}\right]\hat{\imath}$$

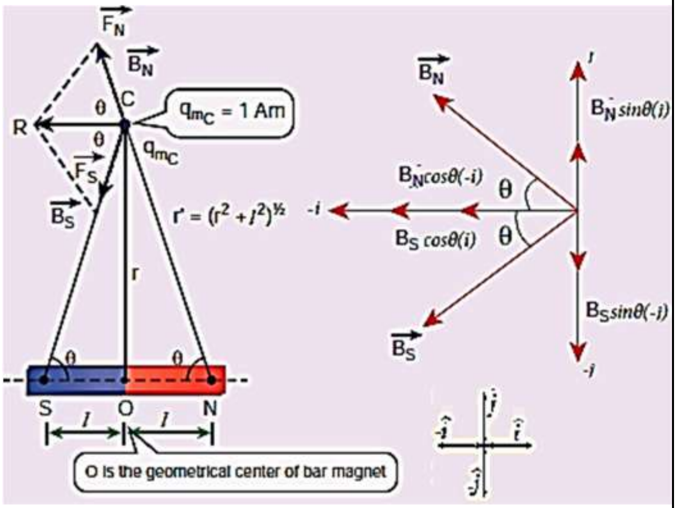
$$\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{2p_m}{r^3}\right]\hat{\imath} \qquad [\because\ p_m = q_m 2l]$$

$$\vec{B}_{axis} = \frac{\mu_0}{4\pi}\left[\frac{2\vec{p}_m}{r^3}\right] \qquad [\because\ \vec{p}_m = p_m\hat{\imath}]$$

- This net magnetic induction $\vec{B}$ is along the direction of $\vec{p}_m$ (i.e. Along S to N).

2. Obtain the magnetic induction at a point on the equatorial line of a bar magnet.

- Consider a bar magnet NS as shown in Figure. Let N be the North Pole and S be the south pole of the bar magnet, each of pole strength q_m and separated by a distance of 2*l*.
- The magnetic field at a point C lies along the equatorial line of the magnet at a distance from the geometrical center O of the bar magnet can be computed by keeping unit north pole (q_{mc} = 1 A m) at C.



- The magnetic induction (force experienced by the unit north pole) at C due to north pole,

$$B_N = \frac{|\vec{F}_N|}{q_{m_c}} = \frac{\mu_0}{4\pi}\frac{q_m}{(r^2+l^2)} \longrightarrow (1)$$

- The magnetic induction (force experienced by the unit north pole) at C due to south pole,

$$B_S = \frac{|\vec{F}_S|}{q_{m_c}} = \frac{\mu_0}{4\pi}\frac{q_m}{(r^2+l^2)} \longrightarrow (2)$$

- The net magnetic induction,

$$\vec{B} = \vec{B}_N + \vec{B}_S$$

$$\vec{B} = -B_N cos\theta\ \hat{\imath} + \vec{B}_N sin\theta(\hat{\jmath}) - B_S cos\theta\ \hat{\imath} - \vec{B}_S sin\theta\ \hat{\jmath}$$

- From equation(1) & (2), $B_N = B_S$

$$\vec{B} = -B_N cos\theta\ \hat{\imath} + \vec{B}_N \cancel{sin\theta}(\hat{\jmath}) - B_N cos\theta\ \hat{\imath} - \vec{B}_N \cancel{sin\theta}\ \hat{\jmath}$$

$$\vec{B} = -2B_N cos\theta\ \hat{\imath}$$

- Substituting B_N value from equation(1), we get,

$$\vec{B} = -\frac{2\mu_0}{4\pi}\frac{q_m}{\left(r^2+l^2\right)}cos\theta\ \hat{\imath}$$

- From ⊿ONC, $cos\theta = \frac{l}{(r^2+l^2)^{\frac{1}{2}}}$

$$\vec{B} = -\frac{2\mu_0}{4\pi}\frac{q_m}{\left(r^2+l^2\right)}\frac{l}{\left(r^2+l^2\right)^{\frac{1}{2}}}\hat{\imath}$$

- Since r >> l, $(r^2 - l^2)^2 \approx r^4$

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{(q_m 2l)}{r^3}\,\hat{\imath}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{p_m}{r^3}\right]\hat{\imath} \qquad [\because\ p_m = q_m 2l]$$

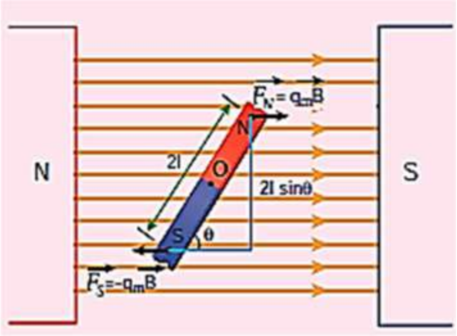
$$\vec{B}_{equatorial} = \frac{\mu_0}{4\pi}\left[\frac{\vec{p}_m}{r^3}\right] \qquad [\because\ \vec{p}_m = p_m\hat{\imath}]$$

- This net magnetic induction $\vec{B}$ is opposite to the direction of $\vec{p}_m$ (i.e. Along N to S).

<u>Note :</u> $\vec{B}_{axis} = 2 \times \vec{B}_{equatorial}$ and $\vec{B}_{axis}$ is opposite to $\vec{B}_{equatorial}$.

3. Compute the torque experienced by a bar magnet in a uniform magnetic field.

- Consider a magnet of length 2l of pole strength q$_m$ kept in a uniform magnetic field $\vec{B}$ as shown in Figure.



- Each pole experiences a force of magnitude q$_m$B but acts in opposite direction. Therefore, the net force exerted on the magnet is zero, so that there is no translatory motion.

- These two forces constitute a couple (about midpoint of bar magnet) which will rotate and try to align in the direction of the magnetic field $\vec{B}$.

$$\vec{\tau} = \overrightarrow{ON} \times \vec{F}_N + \overrightarrow{OS} \times \vec{F}_S$$

$$\vec{\tau} = \overrightarrow{ON} \times (q_m\vec{B}) + \overrightarrow{OS} \times (-q_m\vec{B})$$

- The magnitude of the torque about O is,

$$|\vec{\tau}| = \left|\overrightarrow{ON}\right|\left|q_m\vec{B}\right| \sin\theta + \left|\overrightarrow{OS}\right|\left|-q_m\vec{B}\right| \sin\theta$$

- Since, $\left|\overrightarrow{ON}\right| = \left|\overrightarrow{OS}\right| = l$ & $\left|q_m\vec{B}\right| = \left|-q_m\vec{B}\right| = q_m B$

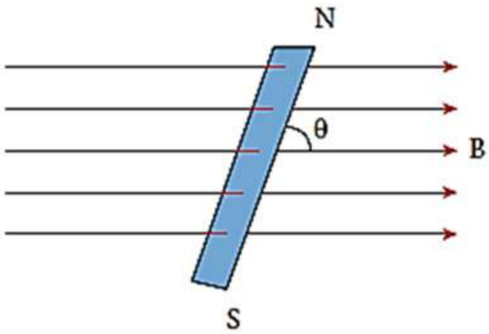
$$\tau = l\, q_m B \sin\theta + l\, q_m B \sin\theta$$
$$\tau = (q_m 2l) B \sin\theta$$
$$\tau = p_m B \sin\theta \qquad [\because\ p_m = q_m 2l]$$

- In vector form,

$$\vec{\tau} = \vec{p}_m \times \vec{B}$$

4. Obtain the expression for potential energy of a bar magnet in a uniform magnetic field.

- When a bar magnet (magnetic dipole) of dipole moment $\vec{p}_m$ is held at an angle θ with the direction of a uniform magnetic field $\vec{B}$ as shown in Figure.



- The magnitude of the torque acting on the dipole is

$$\tau_B = p_m B \sin\theta$$

- If the dipole is rotated through a very small angular displacement dθ against the torque τ_B at constant angular velocity, the work done by external torque τ_{ext} for this small angular displacement is given by,

$$dW = \tau_{ext}\, d\theta$$

- At constant angular velocity, $\tau_{ext} = \tau_B$

$$dW = \tau_B\, d\theta$$
$$dW = p_m B \sin\theta\, d\theta$$

- Total work done in rotating the dipole from θ′ to θ is,

$$W = p_m B \int_{\theta'}^{\theta} \sin\theta\, d\theta$$
$$W = p_m B[-\cos\theta]_{\theta'}^{\theta}$$
$$W = -p_m B(\cos\theta - \cos\theta')$$

- This work done is stored as potential energy in bar magnet. So that,

$$U = W = -p_m B(\cos\theta - \cos\theta')$$

- If the rotation starts from 90^0, $\theta' = 90^0$

$$U = -p_m B\cos\theta$$

- In vector form,

$$U = -\vec{p}_m \cdot \vec{B}$$

- <u>Case 1:</u> If bar magnet is along $\vec{B}$, $\theta = 0^0$

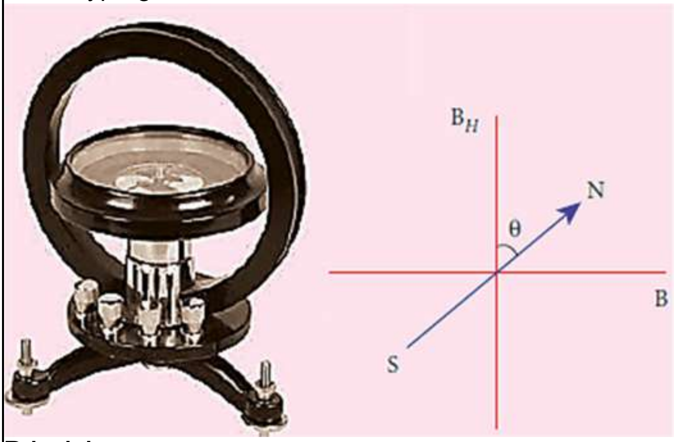
$$U = -p_m B\cos 0^0 = -p_m B = minimum$$

- <u>Case 2:</u> If bar magnet is opposite to $\vec{B}$, $\theta = 180^0$

$$U = -p_m B\cos 180^0 = p_m B = maximum$$

5. Explain the principle, construction and working of a tangent galvanometer.

- ❖ Tangent Galvanometer (Figure) is a device used to measure very small currents. It is a moving magnet type galvanometer.



Principle:

- ❖ **Tangent law**.
  When a magnetic needle or magnet is freely suspended in two mutually perpendicular uniform magnetic fields, it will come to rest in the direction of the resultant of the two fields.

Construction:

- ❖ Tangent Galvanometer (TG) consists of copper coil wounded on a non-magnetic circular frame.
- ❖ The frame is made up of brass or wood which is mounted vertically on a horizontal base table (turn table) with three levelling screws as shown in Figure.
- ❖ The TG is provided with two or more coils of different number of turns say 2 turns, 5 turns and 50 turns which are of different thickness and are used for measuring currents of different strengths.
- ❖ At the center of turn table, a small upright projection is seen on which compass box (also known as magnetometre box) is placed.
- ❖ Compass box consists of a small magnetic needle which is pivoted at the center, such that arrangement shows the center of both magnetic needle and circular coil exactly coincide.
- ❖ A thin aluminium pointer is attached to the magnetic needle normally and moves over circular scale.
- ❖ The circular scale is divided into four quadrants and graduated in degrees which are used to measure the deflection of aluminium pointer on a circular degree scale.
- ❖ In order to avoid parallax error in measurement, a mirror is placed below the aluminium pointer.
- ❖ The circuit connection for Tangent Galvanometer (TG) is done. When no current is passed through the coil, the small magnetic needle lies along horizontal component of Earth's magnetic field.
- ❖ When the circuit is switched ON, the electric current will pass through the circular coil and produce magnetic field.
- ❖ Now there are two fields which are acting mutually perpendicular to each other. They are:
  - The magnetic field (B) due to the electric current in the coil acting normal to the plane of the coil.
  - The horizontal component of Earth's magnetic field (B_H)
- ❖ Because of these crossed fields, the pivoted magnetic needle deflects through an angle θ. From tangent law(figure),

$$B = B_H \tan\theta \longrightarrow (1)$$

- ❖ When an electric current is passed through a circular coil of radius R having N turns, the magnitude of magnetic field at the center is,

$$B = \frac{\mu_0 NI}{2R} \longrightarrow (2)$$

- ❖ From equations(1) and (2), we get,

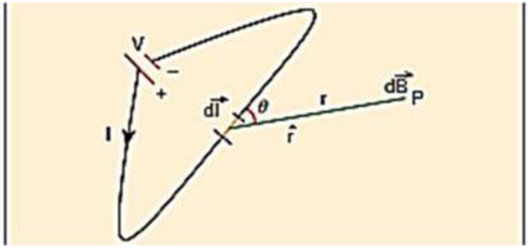
$$\frac{\mu_0 NI}{2R} = B_H \tan\theta$$

- ❖ The horizontal component of Earth's magnetic field can be determined as,

$$B_H = \frac{\mu_0 N}{2R}\left[\frac{I}{\tan\theta}\right]$$

6. State and explain the Biot-Savart's law.

- ❖ **Statement:** The magnitude of magnetic field $\overrightarrow{dB}$ at a point P at a distance r from the small elemental length taken on a conductor carrying current varies
  - directly as the strength of the current I
  - directly as the magnitude of the length element $\overrightarrow{dl}$
  - directly as the sine of the angle (say,θ) between $\overrightarrow{dl}$ and $\hat{r}$.
  - inversely as the square of the distance between the point P and length element $\overrightarrow{dl}$.



$$dB \propto \frac{I\, dl \sin\theta}{r^2}$$

$$dB = k\frac{I\,dl\,sin\theta}{r^2}$$

Where $k = \frac{\mu_0}{4\pi}$ in SI units.

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,sin\theta}{r^2}$$

- In vector form,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\hat{r}}{r^2} - -(3)$$

- Here $d\vec{B}$ is perpendicular to both $I\,\vec{dl}$ and $\hat{r}$.

- The net magnetic field at P due to the conductor is obtained from principle of superposition by considering the contribution from all current elements $I\,\vec{dl}$. Hence integrating equation(3), we get,

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int\frac{I\,\vec{dl}\times\hat{r}}{r^2}$$

- <u>Case 1:</u> If the point P lies on the conductor, $\theta = 0^0$

$$d\vec{B} = 0$$

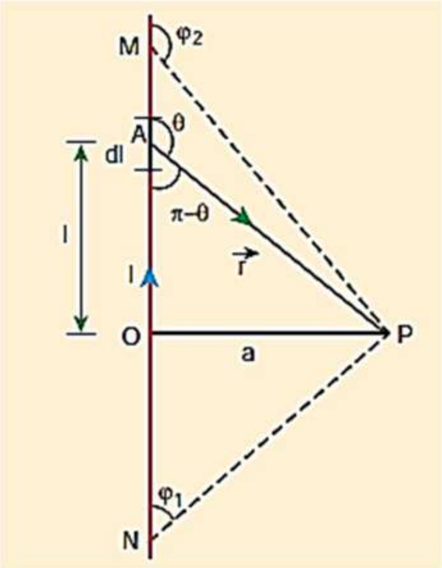
- <u>Case 2:</u> If the point P lies perpendicular to the conductor, $\theta = 90^0$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\hat{n}$$

Where $\hat{n}$ is the unit vector perpendicular to both $I\,\vec{dl}$ and $\hat{r}$.

7. Deduce the relation for the magnetic induction at a point due to an infinitely long straight conductor carrying current.

- Consider a long straight wire NM with current I flowing from N to M as shown in Figure.



- Let P be the point at a distance 'a' from point O. Consider an element of length dl of the wire at a distance l from point O and $\vec{r}$ be the vector joining the element dl with the point P. Let θ be the angle between $\vec{dl}$ and $\vec{r}$.

- Then, the magnetic field at P due to the element is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,dl\,sin\theta}{r^2}\hat{n} \longrightarrow (1)$$

- From ⊿PAO,

$$tan(\pi - \theta) = \frac{a}{l}$$

$$l = \frac{a}{tan(\pi - \theta)} = \frac{a}{-tan\theta} = -a\,cot\theta$$

Differentiating,

$$dl = a\,cosec^2\theta\,d\theta$$

$$and\ from\ ⊿PAO, \quad sin(\pi - \theta) = \frac{a}{r}$$

$$r = \frac{a}{sin(\pi - \theta)} = \frac{a}{sin\,\theta} = a\,cosec\theta$$

- Substituting dl and r value in equation(1), we get,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,(a\,cosec^2\theta\,d\theta)\,sin\theta}{(a\,cosec\theta)^2}\hat{n}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi a}sin\theta\,d\theta\,\hat{n}$$

- This is the magnetic field at a point P due to the current in small elemental length.

- Therefore, the net magnetic field at the point P which can be obtained by integrating $d\vec{B}$ by varying the angle θ from φ₁ to φ₂ is,

$$\vec{B} = \frac{\mu_0 I}{4\pi a}\int_{\varphi_1}^{\varphi_2} sin\theta\,d\theta\,\hat{n}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a}\,[-cos\theta]_{\varphi_1}^{\varphi_2}\ \hat{n}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a}\,(-cos\varphi_2 + cos\varphi_1)\,\hat{n}$$

- For infinetely long straight wire, $\varphi_1 = 0$ and $\varphi_2 = \pi$, the magnetic field is,

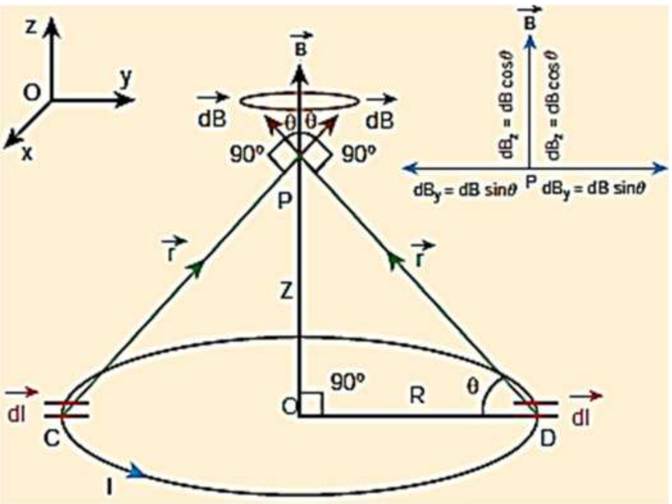
$$\vec{B} = \frac{\mu_0 I}{4\pi a}\,(2)\,\hat{n}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi a}\ \hat{n}$$

- This $\vec{B}$ acts along the line O to P.

8. Obtain a relation for the magnetic induction at a point along the axis of a circular coil carrying current.

- Consider a current carrying circular loop of radius R and let I be the current flowing through the wire in the direction as shown in Figure.



- The magnetic field is found at a point P on the axis of the circular coil at a distance z from its center of the coil O.

- It is computed by taking two diametrically opposite line elements of the coil each of length $\vec{dl}$ at C and D. Let $\vec{r}$ be the vector joining the current element ($I\,\vec{dl}$) at C to the point P.

- According to Biot-Savart's law, the magnetic field at P due to the current element $I\,\vec{dl}$ is,
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\hat{n}$$
Here the angle between $I\,\vec{dl}$ and $\vec{r}$ is 90^0.

- The magnitude of magnetic field due to current element $I\,\vec{dl}$ at C and D are equal because of equal distance from the coil.

- The magnetic field $d\vec{B}$ due to each current element $I\,\vec{dl}$ is resolved into two components; dB sinθ along y - direction and dB cosθ along z - direction.

- Horizontal components of each current element cancels out while the vertical components ($dB\cos\theta\,\hat{k}$) alone contribute to total magnetic field at the point P.

- Now the net magnetic field $\vec{B}$ at point P is,
$$\vec{B} = \int d\vec{B} = \int dB\cos\theta\,\hat{k}$$
$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{dl}{r^2}\cos\theta\,\hat{k}$$

- From figure,
$$PC = PD = r = \sqrt{R^2+Z^2} = (R^2+Z^2)^{\frac{1}{2}}$$
$$\angle CPO = \angle DPO = \theta$$
$$\int dl = 2\pi R$$

- From ΔODA,
$$\cos\theta = \frac{OD}{PD} = \frac{R}{(R^2+Z^2)^{\frac{1}{2}}}$$

- Therefore,
$$\vec{B} = \frac{\mu_0 I}{4\pi}\frac{2\pi R}{(R^2+Z^2)}\frac{R}{(R^2+Z^2)^{\frac{1}{2}}}\hat{k}$$
$$\vec{B} = \frac{\mu_0 \mathrm{I}}{2}\frac{R^2}{(R^2+Z^2)^{\frac{3}{2}}}\hat{k}$$

- This $\vec{B}$ acts along the line O to P.

9. Obtain an expression for magnetic dipole moment of a current loop.

- The magnetic field from the centre of a circular loop of radius R along the axis is given by,
$$\vec{B} = \frac{\mu_0 \mathrm{I}}{2}\frac{R^2}{(R^2+Z^2)^{\frac{3}{2}}}\hat{k}$$

- At larger distance Z >> R, therefore $R^2+Z^2 \approx Z^2$, we have,
$$\vec{B} = \frac{\mu_0 \mathrm{I}}{2}\frac{R^2}{Z^3}\hat{k}$$

- Let A be the area of the circular loop. A = π R^2. So that, we have,
$$\vec{B} = \frac{\mu_0}{2\pi}\frac{\mathrm{I}A}{Z^3}\hat{k} \qquad \left[\because\ R^2 = \frac{A}{\pi}\right]$$

- Multiply and divide by 2, we get,
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\mathrm{I}A}{Z^3}\hat{k} \longrightarrow (1)$$

- The magnetic field at a point along the axis of a bar magnet can be wriiten as,
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\mathrm{p}_m}{Z^3}\hat{k} \longrightarrow (2)$$

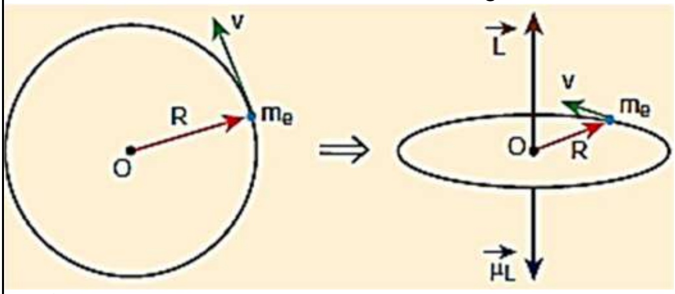
- Comparing the equation(1) and (2), we get,
$$\mathrm{p}_m = IA$$
where p_m is the magnetic dipole moment.

- In a vector form,
$$\vec{\mathrm{p}}_m = I\vec{A}$$

- This implies that a current carrying circular loop behaves as a bar magnet of dipole moment $\vec{\mathrm{p}}_m$.

10. Obtain an expression for magnetic dipole moment of revolving electron.

- Suppose an electron undergoes circular motion around the nucleus as shown in Figure.



- The circulating electron in a loop is like current in a circular loop. The magnetic dipole moment due to current carrying circular loop is,

$$\vec{\mu}_L = I\vec{A}$$

$$or \quad \mu_L = IA \longrightarrow (1)$$

- If T is the time period of an electron, the current due to circular motion of the electron is,

$$\mathrm{I} = \frac{-e}{T} \longrightarrow (2)$$

Here −e is the charge of an electron.

- If R is the radius of the circular orbit and v is the velocity of the electron in the circular orbit, then,

$$\mathrm{T} = \frac{2\pi R}{v} \longrightarrow (3)$$

- Substituting equation(3) in (2), we have,

$$\mathrm{I} = \frac{-ev}{2\pi R} \longrightarrow (4)$$

- Substituting equation(4) in (1), we get,

$$\mu_L = \left(\frac{-ev}{2\pi R}\right) A$$

- But area of the circular loop is A = π R^2. So that,

$$\mu_L = \left(\frac{-ev}{2\pi R}\right) \pi R^2$$

$$\mu_L = -\frac{evR}{2} \longrightarrow (5)$$

- The angular momentum of the electron can be written as,

$$L = pR = mvR$$

Here p = mv is the linear momentum of electron.

$$or \quad vR = \frac{L}{m}$$

- Substituting this in equation(5), we have,

$$\mu_L = -\frac{eL}{2m} \longrightarrow (6)$$

- In vector form,

$$\vec{\mu}_L = -\frac{e\vec{L}}{2m}$$

- The negative sign indicates that the magnetic moment and angular momentum are in opposite direction.

- Taking only magnitude(neglecting –ve sign) of equation(6) and rewritten as,

$$\frac{\mu_L}{L} = \frac{e}{2m} = gyro - magnetic\ ratio$$

Its value is $8.8 \times 10^{10} C\ kg^{-1}$.

- According to Neil's Bohr quantization rule, the angular momentum of an electron moving in a stationary orbit is quantized, which means,

$$L = n\hbar = \frac{nh}{2\pi}$$

- Substituting this in magnitude of equation(6), we get,

$$\mu_L = \frac{neh}{4\pi m}$$

- The minimum magnetic moment can be obtained by substituting n = 1,

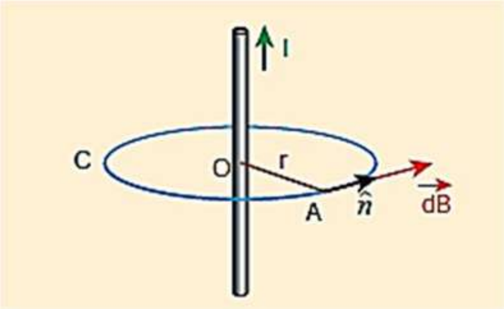
$$(\mu_L)_{min} = \frac{eh}{4\pi m} = \mu_B$$

Where $\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} Am^2$ is called Bohr magneton.

11. Find the magnetic induction due to a long straight conductor using Ampere's circuital law.

- Consider a straight conductor of infinite length carrying current 'I' and the direction of magnetic field as shown in Figure.

- Since the wire is geometrically cylindrical in shape and symmetrical about its axis, we construct an Amperian loop in the form of a circular shape at a distance r from the centre of the conductor as shown in Figure.



- From the Ampere's law, we get

$$\oint_C \vec{B} \cdot \vec{dl} = \mu_0 I$$

$$\oint_C B\ dl\ cos\theta = \mu_0 I$$

- Since $\vec{dl}$ is the line element along the amperian loop (tangent to the circular loop), the angle between $\vec{B}$ and $\vec{dl}$ is zero. (*i.e.* θ = 0^0) Therefore,

$$B\oint_C dl = \mu_0 I$$

- But $\oint_C dl = 2\pi r$, the circumference of the circular loop. So that,

$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

- In vector form,

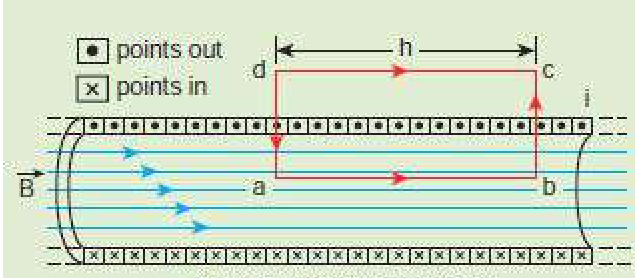
$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{n}$$

Where $\hat{n}$ is the unit vector along the tangent to the Amperian loop.

12. Calculate the magnetic field inside and outside of the long solenoid using Ampere's circuital law.

- Consider a solenoid of length L having N turns. The diametre of the solenoid is assumed to be much smaller when compared to its length and the coil is wound very closely.

- In order to calculate the magnetic field at any point inside the solenoid, consider a Amperian rectangular loop abcd as shown in Figure.



Magnetic field of a solenoid

- Then from Ampere's circuital law,

$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0 I_{encl} \longrightarrow (1)$$

Where I_{encl} is the total current enclosed by the Amperian loop.

- The left hand side of the equation is,

$$\oint_C \vec{B}\cdot\vec{dl} = \oint_a^b \vec{B}\cdot\vec{dl} + \oint_b^c \vec{B}\cdot\vec{dl} + \oint_c^d \vec{B}\cdot\vec{dl} + \oint_d^a \vec{B}\cdot\vec{dl} \longrightarrow (2)$$

- Since $\vec{B}$ and $\vec{dl}$ are perpendicular to each other in the path of the elemental lengths bc and da,

$$\oint_b^c \vec{B}\cdot\vec{dl} = \oint_d^a \vec{B}\cdot\vec{dl} = \oint_d^a B\,dl\,cos90^0 = 0$$

- Since the magnetic field outside the solenoid is zero(i.e. B=0),

$$\oint_C^d \vec{B}\cdot\vec{dl} = 0$$

- Therefore from equation(2),

$$\oint_C \vec{B}\cdot\vec{dl} = \oint_a^b B\,dl\,cos0^0 + 0 + 0 + 0$$

$$\oint_C \vec{B}\cdot\vec{dl} = B\oint_a^b dl$$

- If the length of 'ab' is equal to length of the solenoid, $\oint_a^b dl = h = L$.

$$\oint_C \vec{B}\cdot\vec{dl} = BL \longrightarrow (3)$$

- If I be the current passing through unit turn of the solenoid, for N turns the total current becomes

$$I_{encl} = NI \longrightarrow (4)$$

- Substituting equations(3) & (4) in equation(1), we get,

$$BL = \mu_0 NI$$

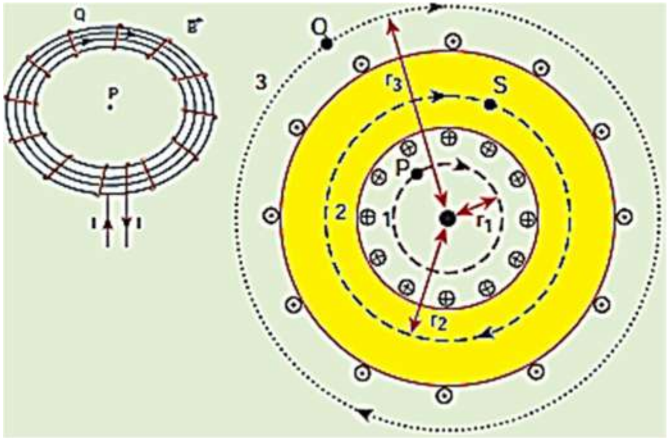
$$B = \mu_0 \frac{N}{L} I$$

- But $n = \frac{N}{L}$, the number of turns per unit length. Therefore,

$$B = \mu_0 nI$$

13. Calculate the magnetic field at interior, exterior and inside of the toroid using Ampere's circuital law.

- Consider a toroid carrying a current I as shown in figure.



- The magnetic field can be calculated at points P, Q and S as below.

(a) Open space interior to the toroid : (At point P)

- Let us calculate the magnetic field B_P at point P. We construct an Amperian loop 1 of radius r_1 around the point P as shown in Figure.
- For simplicity, we take circular loop so that the length of the loop 1 is its circumference.

$$L_1 = 2\pi r_1$$

- Ampere's circuital law for the loop 1 is,

$$\oint_{loop1} \vec{B}_P \cdot \vec{dl} = \mu_0 I_{encl}$$

- Since, the loop1 encloses no current, $I_{encl} = 0$

$$\oint_{loop1} \vec{B}_P \cdot \vec{dl} = 0$$

- This is possible only if the magnetic field at point P vanishes.

$$i.e. \quad \vec{B}_P = 0$$

(b) Open space exterior to the toroid: (At point Q)

- Let us calculate the magnetic field B_Q at point Q. We construct an Amperian loop 3 of radius r_3 around the point Q as shown in Figure.
- The length of the loop 3 is,

$$L_3 = 2\pi r_3$$

- Ampere's circuital law for the loop 3 is,

$$\oint_{loop3} \vec{B}_Q \cdot \vec{dl} = \mu_0 I_{encl}$$

- Since in each turn of the toroid loop, current coming out of the plane of paper is cancelled by the current going into the plane of paper, $I_{encl} = 0$.

$$\oint_{loop3} \vec{B}_Q \cdot \vec{dl} = 0$$

- This is possible only if the magnetic field at point P vanishes.

$$i.e. \quad \vec{B}_Q = 0$$

(c) Inside the toroid: (At point S)

- Let us calculate the magnetic field B_S at point S by constructing an Amperian loop 2 of radius r_2 around the point S as shown in Figure.
- The length of the loop 2 is,

$$L_2 = 2\pi r_2$$

- Ampere's circuital law for the loop 2 is,

$$\oint_{loop2} \vec{B}_S \cdot \vec{dl} = \mu_0 I_{encl} \longrightarrow (1)$$

- Let I be the current passing through the toroid and N be the number of turns of the toroid, then.
$I_{encl} = NI$

$$\oint_{loop2} \vec{B}_S \cdot \vec{dl} = \oint_{loop2} B_S \, dl \cos 0^0 = B_S (2\pi r_2)$$

$$\left[\because \oint_{loop2} dl = 2\pi r_2 \right]$$

- Substituting these values in equation(1), we get,

$$B_S (2\pi r_2) = \mu_0 N I$$

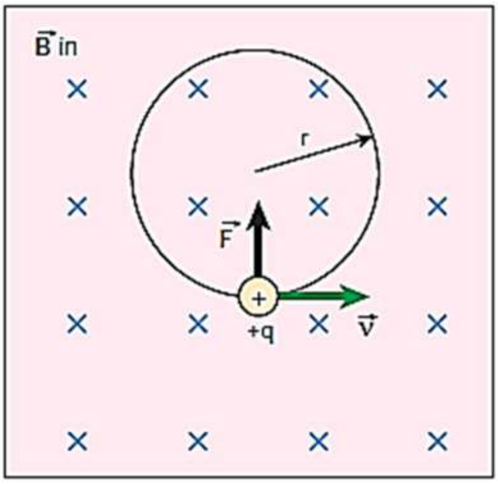
$$B_S = \frac{\mu_0 N I}{2\pi r_2}$$

- But the number of turns per unit length is $n = \frac{N}{2\pi r_2}$, then the magnetic field at point S is,

$$B_S = \mu_0 n I$$

⑭ **Obtain the expressions for time period, frequency and the angular frequency of a charged particle moving in an uniform magnetic field.**

- Consider a charged particle of charge q having mass m enters into a region of uniform magnetic field $\vec{B}$ with velocity $\vec{v}$ such that velocity is perpendicular to the magnetic field.
- As soon as the particle enters into the field, Lorentz force acts on it in a direction perpendicular to both magnetic field $\vec{B}$ and velocity $\vec{v}$. As a result, the charged particle moves in a circular orbit as shown in Figure.



- The Lorentz force on the charged particle is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

- Since Lorentz force alone acts normal to both $\vec{B}$ and $\vec{v}$, the magnitude of the net force on the particle is,

$$\sum_i F_i = F_m = qvB\,sin90^0 = qvB$$

- This Lorentz force acts as centripetal force for the particle to execute circular motion. Therefore,

$$qvB = \frac{mv^2}{r}$$

- The radius of the circular path is,

$$r = \frac{mv}{qB} \dashrightarrow (1)$$

$$r = \frac{p}{qB}$$

Where p = mv is the magnitude of the linear momentum of the particle.

- Let T be the time taken by the particle to finish one complete circular motion(time period), then

$$T = \frac{2\pi r}{v} \dashrightarrow (2)$$

- Substituting equation(1) in (2), we get,

$$T = \frac{2\pi}{v}\left(\frac{mv}{qB}\right)$$

$$T = \frac{2\pi m}{qB} \dashrightarrow (3)$$

This equation is called the cyclotron period.

- The reciprocal of time period is the frequency f, which is,

$$f = \frac{1}{T}$$

$$f = \frac{qB}{2\pi m} \dashrightarrow (4)$$

This equation is called the cyclotron frequency.

- In terms of angular frequency ω,

$$\omega = 2\pi f = 2\pi \frac{qB}{2\pi m}$$

$$\omega = \frac{qB}{2m} \dashrightarrow (5)$$

This equation is called the cyclotron gyro-frequency.

- From equations(3),(4) and (5), we infer that time period, frequency and angualr frequency of the charged particle depend only on charge to mass ratio but not on the velocity and radius of the circular path.

⑮ Explain the principle, construction and working of a cyclotron. Hence discuss its limitations.

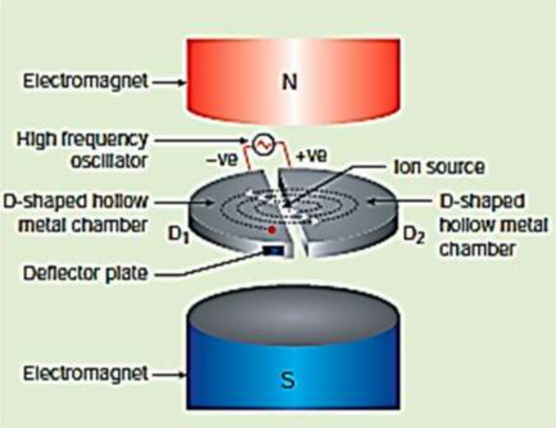
- Cyclotron is a device used to accelerate the charged particles to gain large kinetic energy. It is also called as high energy accelerator.

Principle:

- When a charged particle moves normal to the magnetic field, it experiences magnetic Lorentz force.

Construction:

- The schematic diagram of a cyclotron is shown in Figure.



- The particles are allowed to move in between two semicircular metal containers called Dees (hollow D - shaped objects).

- Dees are enclosed in an evacuated chamber and it is kept in a region with uniform magnetic field controlled by an electromagnet.

- The direction of magnetic field is normal to the plane of the Dees.

- The two Dees are kept separated with a gap and the source S (which ejects the particle to be accelerated) is placed at the center in the gap between the Dees.

- Dees are connected to high frequency alternating potential difference.

Working:

- Let us assume that the ion ejected from source S is positively charged. As soon as ion is ejected, it is accelerated towards a Dee (say, Dee – 1) which has negative potential at that time.

- Since the magnetic field is normal to the plane of the Dees, the ion undergoes circular path.

- After one semi-circular path in Dee-1, the ion reaches the gap between Dees. At this time, the polarities of the Dees are reversed so that the ion is now accelerated towards Dee-2 with a greater velocity.

- For this circular motion, the centripetal force of the charged particle q is provided by Lorentz force.

$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB}$$

$$or \qquad r \propto v$$

- From the above equation, we know that the increase in velocity increases the radius of circular path.

- This process continues and hence the particle undergoes spiral path of increasing radius. Once it reaches near the edge, it is taken out with the help of deflector plate and allowed to hit the target T.

- The frequency at which the positive ion circulates in the magnetic field can be written as,

$$f = \frac{qB}{2\pi m}$$

- However, to operate Cyclotron, this frequency ‘f’ must be equal to electrical oscillator frequency f_{osc}. This is called resonance condition. Therefore,

$$f_{osc} = f = \frac{qB}{2\pi m}$$

- The time period of oscillation is,

$$T = \frac{1}{f_{osc}} = \frac{2\pi m}{qB}$$

- Since the velocity of the charged particle is,

$$v = r\omega = r\left(\frac{2\pi}{T}\right) = r\frac{2\pi\, qB}{2\pi m} = \frac{qBr}{m}$$

- The kinetic energy of the charged particle becomes,

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qBr}{m}\right)^2$$

$$KE = \frac{q^2B^2r^2}{2m}$$

Limitations of cyclotron:

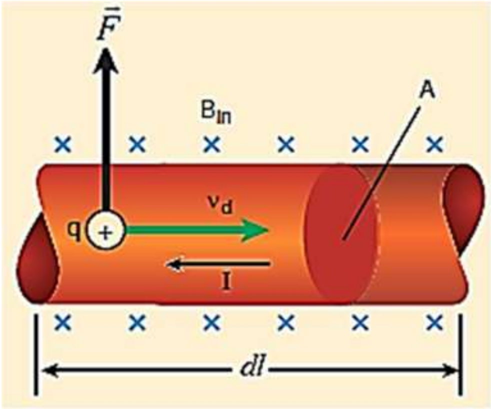
- The speed of the ion is limited .

- Electron cannot be accelerated.

- Uncharged particles cannot be accelerated.

⑯ Obtain an expression for force on a current carrying conductor placed in a magnetic field.

- Consider a small segment of wire of length d*l*, with cross- sectional area A and current I as shown in Figure.



- The free electrons drift opposite to the direction of current. So the relation between current I and drift velocity v_d can be written as,

$$I = -neA\vec{v}_d$$

- Now the current element can be written as,

$$I\,\vec{dl} = -neA\,dl\,\vec{v}_d \longrightarrow (1)$$

- If the wire segment is kept in a magnetic field $\vec{B}$, then average force experienced by the single electron in the wire is,

$$d\vec{F} = -e\left(\vec{v}_d \times \vec{B}\right)$$

- Suppose the wire segment consists of N number of free electrons, the Lorentz force becomes,

$$d\vec{F} = -Ne\left(\vec{v}_d \times \vec{B}\right) \longrightarrow (2)$$

- If n is the number of free electrons per unit volume, the number of free electrons in the small element of volume V=Ad*l* is given by,

$$N = nV = n\,A\,dl \qquad \left[\because\ n = \frac{N}{V}\right]$$

- Substituting this N value in equation(2), we get,

$$d\vec{F} = -n\,A\,dl\,e\left(\vec{v}_d \times \vec{B}\right)$$
$$d\vec{F} = -n\,Ae\,dl\,e\left(\vec{v}_d \times \vec{B}\right)$$

- Substituting equation(1), we have,

$$d\vec{F} = \left(I\,\vec{dl} \times \vec{B}\right)$$

- For a conducting wire of length *l* , the force can be given by,

$$\vec{F} = \left(I\,\vec{l} \times \vec{B}\right)$$

- In magnitude,

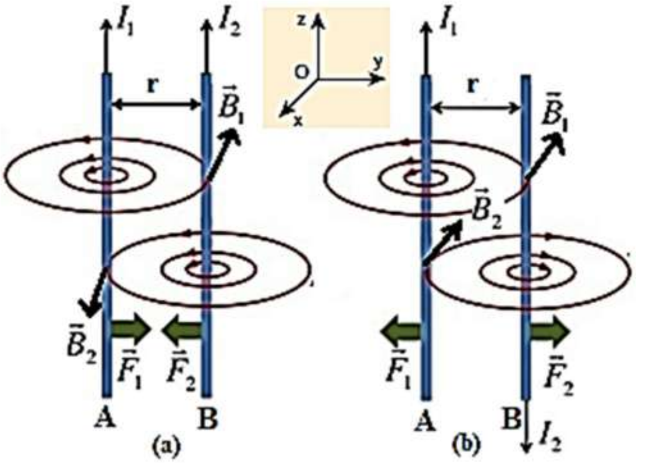
$$F = BIl\,sin\theta$$

- <u>**Case 1:**</u> If the wire is placed along the direction of the magnetic field i.e. $\theta = 0^0$, F = 0.

- <u>**Case 2:**</u> If the wire is placed perpendicular to the direction of the magnetic field i.e. $\theta = 90^0$, F = BI*l*.

⑰ Find the force between two long parallel current carrying conductors.

- ❖ Consider two long straight parallel current carrying conductors A and B in air are separated by a distance r as shown in Figure.



- ❖ Let I_1 and I_2 be the electric currents passing through the conductors A and B in same direction (i.e. along z - direction) respectively.
- ❖ The net magnetic field at a distance r due to current I_1 in conductor A is,

$$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi r}\hat{\imath}$$

- ❖ From thumb rule, the direction of this magnetic field is perpendicular to the plane of the paper and inwards. i.e. along negative $\hat{\imath}$ direction.
- ❖ The Lorentz force F_2 acting on the conductor B due to magnetic field $\vec{B}_1$ can be written as,

$$\vec{F}_2 = \left(I_2\vec{l} \times \vec{B}_1\right)$$

- ❖ But $I_2\vec{l} = I_2 l\,\hat{k}$ and $\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi r}\hat{\imath}$, Therefore,

$$\vec{F}_2 = \left(I_2 l\,\hat{k} \times -\frac{\mu_0 I_1}{2\pi r}\hat{\imath}\right)$$
$$\vec{F}_2 = -\frac{\mu_0 I_1 I_2 l}{2\pi r}\left(\hat{k} \times \hat{\imath}\right)$$
$$\vec{F}_2 = -\frac{\mu_0 I_1 I_2 l}{2\pi r}\hat{\jmath} \dashrightarrow (1)$$

This force is acting towards the conductor A.

- ❖ Now force per unit length on the conductor B,

$$\frac{\vec{F}_2}{l} = -\frac{\mu_0 I_1 I_2}{2\pi r}\hat{\jmath}$$

- ❖ Similarly, the net magnetic field at a distance r due to current I_2 in conductor B is,

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi r}\hat{\imath}$$

- ❖ From thumb rule, the direction of this magnetic field is perpendicular to the plane of the paper and outwards. i.e. along positive $\hat{\imath}$ direction.
- ❖ The Lorentz force F_1 acting on the conductor A due to magnetic field $\vec{B}_2$ can be written as,

$$\vec{F}_1 = \left(I_1\vec{l} \times \vec{B}_2\right)$$

- ❖ But $I_1\vec{l} = I_1 l\,\hat{k}$ and $\vec{B}_2 = \frac{\mu_0 I_2}{2\pi r}\hat{\imath}$, Therefore,

$$\vec{F}_1 = \left(I_1 l\,\hat{k} \times \frac{\mu_0 I_2}{2\pi r}\hat{\imath}\right)$$
$$\vec{F}_1 = \frac{\mu_0 I_1 I_2 l}{2\pi r}\left(\hat{k} \times \hat{\imath}\right)$$
$$\vec{F}_1 = \frac{\mu_0 I_1 I_2 l}{2\pi r}\hat{\jmath} \dashrightarrow (2)$$

This force is acting towards the conductor B.

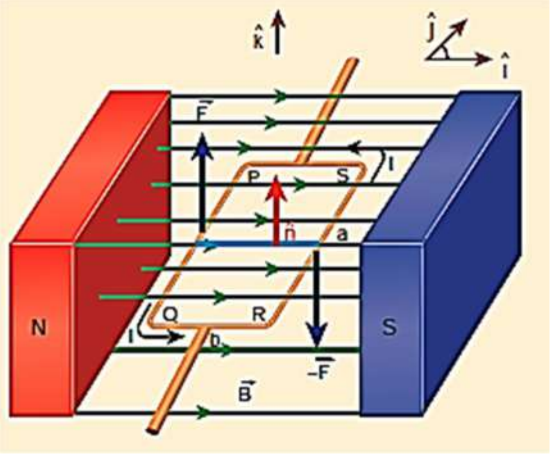
- ❖ Now force per unit length on the conductor A,

$$\frac{\vec{F}_1}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}\hat{\jmath}$$

- ❖ Thus from equations (1) and (2), the force experienced by two parallel current carrying conductors is,
  - attractive if the direction of electric current passing through them is same.(Figure (a))
  - repulsive if the direction of electric current passing through them is opposite.(Figure (b))

18. Obtain an expression for torque on a current loop when its unit vector is perpendicular to the magnetic field.

- ❖ Consider a single rectangular loop PQRS lying in xy plane, kept in a uniform magnetic field $\vec{B}$ acting along the x direction as shown in figure.



- ❖ Let a and b be the length and breadth of the rectangular loop respectively and $\hat{n}$ be the unit vector normal to the plane of the current loop.
- ❖ When an electric current is sent through the loop, the net force acting is zero but there will be net torque acting on it.
- ❖ We shall now consider unit vector $\hat{n}$ pointing perpendicular to the field.

- Let the loop be divided into four sections PQ, QR, RS and SP. The Lorentz force on each loop can be calculated as follows:

(a) Force on section PQ:

$$\vec{l} = -a\hat{j} \quad and \quad \vec{B} = B\hat{\imath}$$

$$\vec{F}_{PQ} = I\vec{l} \times \vec{B}$$
$$\vec{F}_{PQ} = -IaB(\hat{j} \times \hat{\imath}) = -IaB(-\hat{k})$$
$$\vec{F}_{PQ} = IaB\hat{k}$$

Hence this force acts along the direction of $\hat{k}$

(b) Force on section QR:

$$\vec{l} = b\hat{\imath} \quad and \quad \vec{B} = B\hat{\imath}$$

$$\vec{F}_{QR} = I\vec{l} \times \vec{B}$$
$$\vec{F}_{QR} = IbB(\hat{\imath} \times \hat{\imath}) = \vec{0}$$

Hence the there is no force acting on this segment.

(c)Force on section RS:

$$\vec{l} = a\hat{j} \quad and \quad \vec{B} = B\hat{\imath}$$

$$\vec{F}_{RS} = I\vec{l} \times \vec{B}$$
$$\vec{F}_{RS} = IaB(\hat{j} \times \hat{\imath}) = IaB(-\hat{k})$$
$$\vec{F}_{RS} = -IaB\hat{k}$$

Hence this force acts opposite the direction of $\hat{k}$

(d) Force on section SP:

$$\vec{l} = -b\hat{\imath} \quad and \quad \vec{B} = B\hat{\imath}$$

$$\vec{F}_{SP} = I\vec{l} \times \vec{B}$$
$$\vec{F}_{SP} = -IbB(\hat{\imath} \times \hat{\imath}) = \vec{0}$$

Hence the there is no force acting on this segment.

- The net force on the rectangular loop is,

$$\vec{F}_{net} = \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RS} + \vec{F}_{SP}$$
$$\vec{F}_{net} = IaB\hat{k} + \vec{0} - IaB\hat{k} + \vec{0} = \vec{0}$$

- Hence, the net force on the rectangular loop in this configuration is zero.

- Now let us calculate the net torque due to these forces about an axis passing through the center.

$$\vec{\tau}_{net} = \vec{\tau}_{PQ} + \vec{\tau}_{QR} + \vec{\tau}_{RS} + \vec{\tau}_{SP} \longrightarrow (1)$$

$$\vec{\tau}_{PQ} = \vec{r}_{PQ} \times \vec{F}_{PQ} = \frac{b}{2}\, IaB\left(-\hat{\imath} \times \hat{k}\right) = \frac{b}{2}\, IaB\hat{j}$$
$$\vec{\tau}_{QR} = \vec{r}_{QR} \times \vec{F}_{QR} = \frac{a}{2}\,\left(-\hat{j} \times \vec{0}\right) = 0\,\hat{j}$$
$$\vec{\tau}_{RS} = \vec{r}_{RS} \times \vec{F}_{RS} = \frac{b}{2} IaB\left(\hat{\imath} \times -\hat{k}\right) = \frac{b}{2}\, IaB\hat{j}$$
$$\vec{\tau}_{SP} = \vec{r}_{SP} \times \vec{F}_{SP} = \frac{a}{2}\,\left(\hat{j} \times \vec{0}\right) = 0\,\hat{j}$$

- Substituting these values in equation(1), we get,

$$\vec{\tau}_{net} = \left(\frac{b}{2}\, IaB + 0 + \frac{b}{2}\, IaB + 0\right)\hat{j}$$
$$\vec{\tau}_{net} = IabB\hat{j}$$

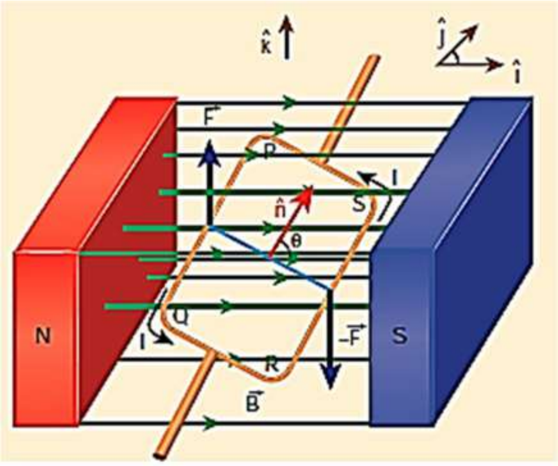
- Since, A = ab is the area of the rectangular loop PQRS, the net torque for this configuration is,

$$\vec{\tau}_{net} = IAB\hat{j}$$

- When the loop starts rotating about y axis due to this torque, the magnetic field $\vec{B}$ is no longer in the plane of the loop. So the above equation is the special case.

19. Obtain an expression for torque on a current loop when its unit vector is at an angle with the magnetic field.

- Consider a single rectangular loop PQRS whose unit vector $\hat{n}$ makes an angle θ with the uniform magnetic field $\vec{B}$ acting along the x direction as shown in figure.



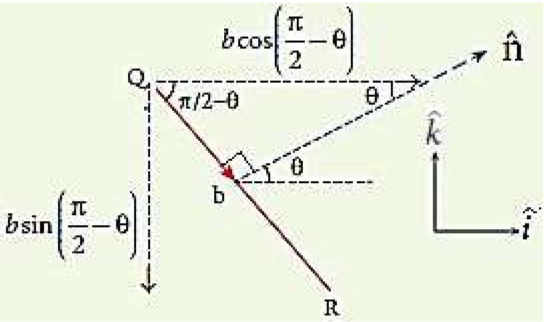
(a) Force on section PQ:

$$\vec{l} = -a\hat{j} \quad and \quad \vec{B} = B\hat{\imath}$$

$$\vec{F}_{PQ} = I\vec{l} \times \vec{B}$$
$$\vec{F}_{PQ} = -IaB(\hat{j} \times \hat{\imath}) = -IaB(-\hat{k})$$
$$\vec{F}_{PQ} = IaB\hat{k}$$

Hence this force acts along the direction of $\hat{k}$

(b) Force on section QR:



$$\vec{l} = b\cos\left[\frac{\pi}{2} - \theta\right]\hat{\imath} - b\sin\left[\frac{\pi}{2} - \theta\right]\hat{k}$$
$$\vec{B} = B\hat{\imath}$$

$$\vec{F}_{QR} = I\vec{l}\times\vec{B}$$

$$\vec{F}_{QR} = IbBcos\left[\frac{\pi}{2}-\theta\right](\hat{\imath}\times\hat{\imath}) - IbBsin\left[\frac{\pi}{2}-\theta\right](\hat{k}\times\hat{\imath})$$

$$\vec{F}_{QR} = -IbBsin\left[\frac{\pi}{2}-\theta\right]\hat{\jmath} \qquad \left[\because \begin{matrix}\hat{\imath}\times\hat{\imath}=0\\ \hat{k}\times\hat{\imath}=\hat{\jmath}\end{matrix}\right]$$

$$\vec{F}_{QR} = -IbBcos\theta\hat{\jmath}$$

Hence this force acts opposite to the direction of $\hat{\jmath}$.

(c)Force on section RS:

$$\vec{l} = a\hat{\jmath} \quad and \quad \vec{B} = B\hat{\imath}$$

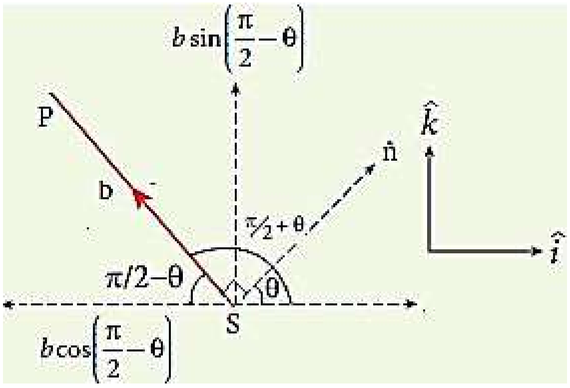
$$\vec{F}_{RS} = I\vec{l}\times\vec{B}$$

$$\vec{F}_{RS} = IaB(\hat{\jmath}\times\hat{\imath}) = IaB(-\hat{k})$$

$$\vec{F}_{RS} = -IaB\hat{k}$$

Hence this force acts opposite the direction of $\hat{k}$

(d) Force on section SP:



$$\vec{l} = -bcos\left[\frac{\pi}{2}-\theta\right]\hat{\imath} + bsin\left[\frac{\pi}{2}-\theta\right]\hat{k}$$

$$\vec{B} = B\hat{\imath}$$

$$\vec{F}_{SP} = I\vec{l}\times\vec{B}$$

$$\vec{F}_{SP} = -IbBcos\left[\frac{\pi}{2}-\theta\right](\hat{\imath}\times\hat{\imath}) + IbBsin\left[\frac{\pi}{2}-\theta\right](\hat{k}\times\hat{\imath})$$

$$\vec{F}_{SP} = IbBsin\left[\frac{\pi}{2}-\theta\right]\hat{\jmath} \qquad \left[\because \begin{matrix}\hat{\imath}\times\hat{\imath}=0\\ \hat{k}\times\hat{\imath}=\hat{\jmath}\end{matrix}\right]$$

$$\vec{F}_{SP} = IbBcos\theta\hat{\jmath}$$

Hence this force acts along the direction of $\hat{\jmath}$.

- The net force on the rectangular loop is,

$$\vec{F}_{net} = \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RS} + \vec{F}_{SP}$$

$$\vec{F}_{net} = IaB\hat{k} - IbBcos\theta\hat{\jmath} - IaB\hat{k} + IbBcos\theta\hat{\jmath}$$

$$\vec{F}_{net} = \vec{0}$$

- Hence, the net force on the rectangular loop in this configuration is zero.

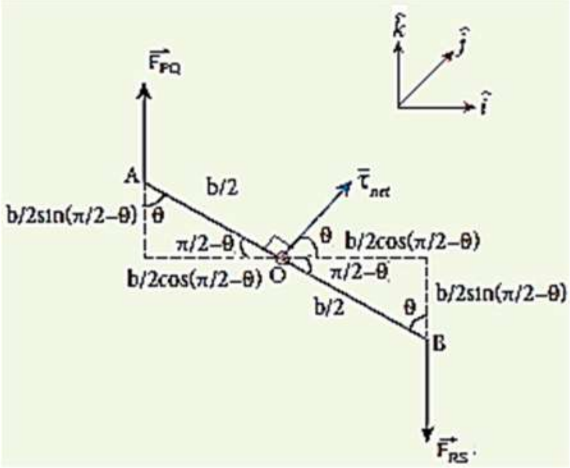
- Now let us calculate the net torque due to these forces about an axis passing through the center.

$$\vec{\tau}_{net} = \vec{\tau}_{PQ} + \vec{\tau}_{QR} + \vec{\tau}_{RS} + \vec{\tau}_{SP} \longrightarrow (1)$$

- Even though the forces on QR and SP are equal and opposite, it can not produce any torque since loop is regid in this direction.

$$i.e. \quad \vec{\tau}_{QR} = \vec{\tau}_{SP} = \vec{0} --(2)$$

- But torque due to PQ and RS can be arrived as,



- From the above diagram,

$$\overrightarrow{OA} = \frac{b}{2}cos\left(\frac{\pi}{2}-\theta\right)(-\hat{\imath}) + \frac{b}{2}sin\left(\frac{\pi}{2}-\theta\right)\hat{k}$$

$$\overrightarrow{OA} = \frac{b}{2}\left(-sin\theta\,\hat{\imath} + cos\theta\,\hat{k}\right)$$

$$\overrightarrow{OB} = \frac{b}{2}cos\left(\frac{\pi}{2}-\theta\right)\hat{\imath} + \frac{b}{2}sin\left(\frac{\pi}{2}-\theta\right)(-\hat{k})$$

$$\overrightarrow{OB} = \frac{b}{2}\left(sin\theta\,\hat{\imath} - cos\theta\,\hat{k}\right)$$

- Therefore,

$$\vec{\tau}_{PQ} = \overrightarrow{OA}\times\vec{F}_{PQ}$$

$$\vec{\tau}_{PQ} = \frac{b}{2}\left(-sin\theta\,\hat{\imath} + cos\theta\,\hat{k}\right)\times IaB\,\hat{k}$$

$$\vec{\tau}_{PQ} = \frac{1}{2}IabB\,sin\theta\,\hat{\jmath} \longrightarrow (3) \quad \left[\because \begin{matrix}\hat{\imath}\times\hat{k}=-\hat{\jmath}\\ \hat{k}\times\hat{k}=0\end{matrix}\right]$$

and

$$\vec{\tau}_{RS} = \overrightarrow{OB}\times\vec{F}_{RS}$$

$$\vec{\tau}_{RS} = \frac{b}{2}\left(sin\theta\,\hat{\imath} - cos\theta\,\hat{k}\right)\times\left(-IaB\,\hat{k}\right)$$

$$\vec{\tau}_{RS} = \frac{1}{2}IabB\,sin\theta\,\hat{\jmath} \longrightarrow (4) \quad \left[\because \begin{matrix}\hat{\imath}\times\hat{k}=-\hat{\jmath}\\ \hat{k}\times\hat{k}=0\end{matrix}\right]$$

- Substituting equations(2),(3) and(4) in(1), we have,

$$\vec{\tau}_{net} = \frac{1}{2}IabB\,sin\theta\,\hat{\jmath} + \vec{0} + \frac{1}{2}IabB\,sin\theta\,\hat{\jmath} + \vec{0}$$

$$\vec{\tau}_{net} = IabB\,sin\theta\,\hat{\jmath}$$

- Since, A = ab is the area of the rectangular loop PQRS, the net torque for this configuration is,

$$\vec{\tau}_{net} = IAB\,sin\theta\,\hat{\jmath}$$

$$\boldsymbol{\vec{\tau}_{net} = p_m B\,sin\theta\,\hat{\jmath} = \vec{p}_m\times\vec{B}}$$

Where $p_m = IA$ is the magnetic dipole moment.

- **Case 1:** If $\theta = 90^0$,

  $$\vec{\tau}_{net} = IabB = p_m B\,\hat{j} \Longrightarrow maximum$$

  Here $\vec{p}_m$ is perpendicular to $\vec{B}$.

- **Case 2:** If $\theta = 0^0$ or 180^0,

  $$\vec{\tau}_{net} = \vec{0} \Longrightarrow minimum$$

  For $\theta = 0^0$, $\vec{p}_m$ is parallel to $\vec{B}$ and for $\theta = 180^0$, $\vec{p}_m$ is anti-parallel to $\vec{B}$.

20. Explain the principle, construction and working of a moving coil galvanometer.

- Moving coil galvanometer is a device which is used to indicate the flow of current in an electrical circuit.

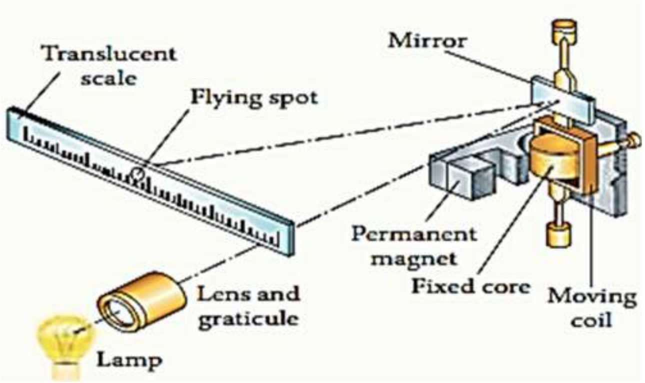
Principle:

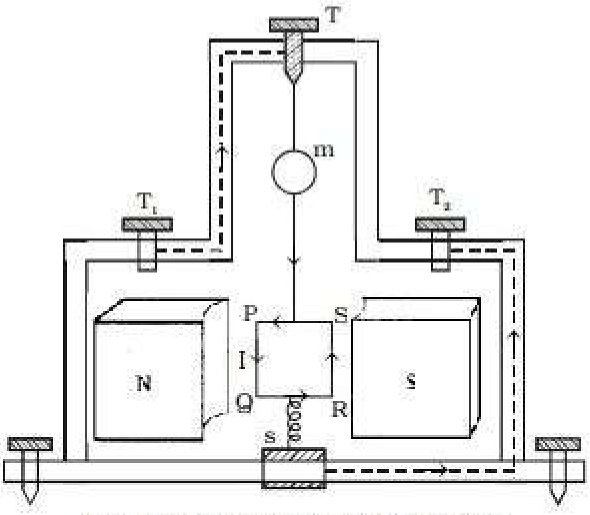
- When a current carrying loop is placed in a uniform magnetic field it experiences a torque.

Construction :

- A moving coil galvanometer consists of a rectangular coil PQRS of insulated thin copper wire.

- The coil contains a large number of turns wound over a light metallic frame. A cylindrical soft-iron core is placed symmetrically inside the coil as shown in Figure.





Moving coil galvanometer

- The rectangular coil is suspended freely between two pole pieces of a horse-shoe magnet.

- The upper end of the rectangular coil is attached to one end of fine strip of phosphor bronze and the lower end of the coil is connected to a hair spring which is also made up of phosphor bronze.

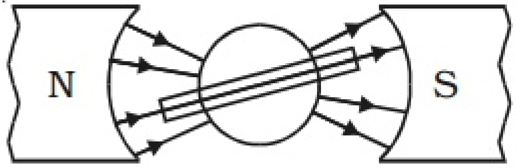
- In a fine suspension strip W, a small plane mirror is attached in order to measure the deflection of the coil with the help of lamp and scale arrangement.

- The other end of the mirror is connected to a torsion head T. In order to pass electric current through the galvanometer, the suspension strip W and the spring S are connected to terminals.

Working:

- Consider a single turn of the rectangular coil PQRS whose length be l and breadth b. PQ = RS = l and QR = SP = b.

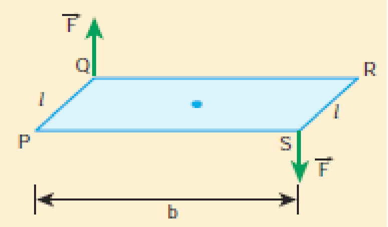
- Let I be the electric current flowing through the rectangular coil PQRS as shown in Figure. The horse-shoe magnet has hemi - spherical magnetic poles which produces a radial magnetic field.



Radial magnetic field

- Due to this radial field, the sides QR and SP are always parallel to the magnetic field 'B' and experience no force.

- The sides PQ and RS are always perpendicular to the magnetic field 'B' and experience equal and opposite force respectively. Such that, torque is produced.



- For single turn, the deflection couple can be written as,

  $$\tau = bF = bBIl = (lb)BI = ABI$$

  Here A = l b is the area of the coil.

- For coil with N turns,

  $$\tau = NABI \longrightarrow (1)$$

- Due to this deflecting torque, the coil gets twisted and restoring torque (also known as restoring couple) is developed.

- Hence the magnitude of restoring couple is proportional to the amount of twist θ.Thus,

$$\tau = K\theta \dashrightarrow (2)$$

where K is the restoring couple per unit twist or torsional constant of the spring.

- At equilibrium, the deflection couple is equal to the restoring couple. Therefore equating (1) and (2), we get,

$$NABI = K\theta$$

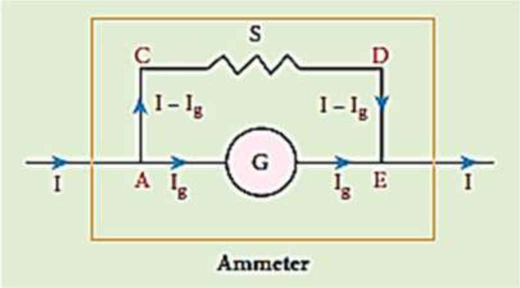
$$I = \frac{K}{NAB}\theta$$

$$I = G\theta$$

where, $G = \frac{K}{NAB}$ is called galvanometer constant or current reduction factor of the galvanometer.

21) Discuss the conversion of galvanometer into an ammeter.

- Ammeter is an instrument used to measure current flowing in the electrical circuit.
- A galvanometer is converted into an ammeter by connecting a low resistance in parallel with the galvanometer.
- This low resistance is called shunt resistance S. The scale is now calibrated in ampere and the range of ammeter depends on the values of the shunt resistance.



- Let I be the current passing through the circuit as shown in Figure. When current I reaches the junction A, it divides into two components.
- Let I_g be the current passing through the galvanometer of resistance R_g through a path AGE and the remaining current $(I - I_g)$ passes along the path ACDE through shunt resistance S.
- The value of shunt resistance is so adjusted that current I_g produces full scale deflection in the galvanometer.
- The potential difference across galvanometer is same as the potential difference across shunt resistance.

$$V_{galvanometer} = V_{shunt}$$

$$I_g R_g = (I - I_g)S$$

$$S = \frac{I_g}{(I - I_g)} R_g$$

$$or \qquad I_g = \frac{S}{(S + R_g)} I$$

$$I_g \propto I$$

- Since the deflection in the galvanometer is proportional to the current passing through it.

$$\theta \propto I_g \propto I \qquad \left[\because \theta = \frac{1}{G} I_g\right]$$

- So, the deflection in the galvanometer measures the current I passing through the circuit (ammeter).
- Shunt resistance is connected in parallel to galvanometer. Therefore, resistance of ammeter can be determined by computing the effective resistance, which is

$$\frac{1}{R_{eff}} = \frac{1}{R_g} + \frac{1}{S} = \frac{R_g + S}{R_g S}$$

$$R_{eff} = \frac{R_g S}{R_g + S} = R_a$$

- Since R_g and S are very small, the ammeter resistance is also very small. Such that ammeter is always connected in series to measure the current as it will not change the circuit resistance appreciably.
- The resistance of an ideal ammeter must be equal to zero. So that, $I_{actual} < I_{ideal}$
- Then, the percentage error in measuring a current through an ammeter is,

$$\frac{\Delta I}{I} \times 100\% = \frac{I_{ideal} - I_{actual}}{I_{actual}} \times 100\%$$

- In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is,

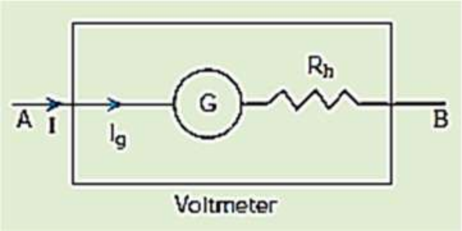
$$S = \frac{G}{n - 1}$$

Where G is the galvanometer constant.

22) Discuss the conversion of galvanometer into an voltmeter.

- A voltmeter is an instrument used to measure potential difference across any two points in the electrical circuits.

- A galvanometer is converted into a voltmeter by connecting high resistance R_h in series with galvanometer as shown in Figure.



- Let R_g be the resistance of galvanometer and I_g be the current with which the galvanometer produces full scale deflection.

- Since the galvanometer is connected in series with high resistance, the current in the electrical circuit is same as the current passing through the galvanometer.

$$I = I_g$$

$$I = I_g = \frac{Potential\ difference}{Total\ resistance}$$

- But from figure, the total resistance, $R_v = R_g + R_h$. Therefore,

$$I_g = \frac{V}{R_g + R_h} \longrightarrow (1)$$

$$R_h = \frac{V}{I_g} - R_g$$

- From equation(1), we can note that, $I_g \propto V$. But the deflection in the galvanometer is proportional to the current I_g. Therefore,

$$\theta \propto I_g \propto V$$

- Since the resistance of voltmeter is very large, it is always connected in parallel with the circuit element to draw least current in the circuit.

- An ideal voltmeter is one which has infinite resistance.

- In order to increase the range of voltmeter n times the value of resistance to be connected in series with galvanometer is R = (n-1) G.

4. Electromagnetic Induction And Alternating Current

1. What is electromagnetic induction?

Whenever the magnetic flux linked with a closed coil changes, an emf is induced and hence an electric current flows in the circuit. This phenomenon is known as electromagnetic induction.

2. Define magnetic flux. Give its unit.

The magnetic flux through an area A in a magnetic field is defined as the number of magnetic field lines passing normally through that area. Its unit is weber(Wb) or Tm^{-2}.

$$\Phi_B = \int_A \vec{B} \cdot d\vec{A} = BA\cos\theta$$

3. State Faraday's law of electromagnetic induction.

- **First law:**

  Whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit.

- **Second law:**

  The magnitude of induced emf in a closed circuit is equal to the rate of change of magnetic flux linked with the circuit.

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

4. State Lenz's law.

Lenz's law states that the direction of the induced current is such that it always opposes the cause (rate of change of magnetic flux) responsible for its production.

5. State Fleming's right hand rule(generator rule).

The thumb, index finger and middle finger of right hand are stretched out in mutually perpendicular directions. If the index finger points the direction of the magnetic field and the thumb indicates the direction of motion of the conductor, then the middle finger will indicate the direction of the induced current.

6. How is Eddy current produced? How do they flow in a conductor?

When magnetic flux linked with a conductor in the form of a sheet or a plate changes, an emf is induced. The induced currents flow in concentric circular paths called Eddy currents or Foucault currents.

7. What for an inductor is used? Give some examples.

Inductor is a device used to store energy in a magnetic field when an electric current flows through it.

<u>Ex:</u> coils, solenoids and toroids.

8. What is self-induction?

If the current flowing through the coil changes, the change in flux takesplace and hence emf is induced in that same coil. This phenomenon is known as self-induction.

9. Define self-inductance or inductance or coefficient of self induction. Give its unit.

Self-inductance(L) of a coil is defined as the opposing emf induced in the coil when the rate of change of current through the coil is 1 A s^{-1}. Its unit is Wb A^{-1} or V s A^{-1} or henry(H).

10. Define 1 henry from self-inductance.

The inductance of the coil is one henry if a current changing at the rate of 1 A s^{-1} induces an opposing emf of 1 V in it.

11. What is mutual inductance?

When an electric current passing through a coil changes with time, an emf is induced in the neighbouring coil. This phenomenon is known as mutual induction.

12. Define mutual inductance or coefficient of mutual induction. Give its unit.

Mutual inductance M_{21} is also defined as the opposing emf induced in the coil 2 when the rate of change of current through the coil 1 is 1 As^{-1}. Its unit is Wb A^{-1} or V s A^{-1} or henry(H).

13. Define 1 henry from mutual inductance.

The mutual inductance between two coils is one henry if a current changing at the rate of $1As^{-1}$ in coil 1 induces an opposing emf of 1V in coil 2.

14. What is electromotive force(emf)? Give its unit.

Electromotive force is the work done in moving unit electric charge around the circuit. Its unit is J C^{-1} or volt.

15. Mention the ways of producing induced emf.

- By changing the magnetic field B.
- By changing the area A of the coil.
- By changing the relative orientation θ of the coil with magnetic field.

16. What is AC generator or alternator? State its principle.

- AC generator or alternator is an energy conversion device. It converts mechanical energy used to rotate the coil or field magnet into electrical energy.
- <u>Principle:</u> Electromagnetic induction.

17. What are the advantages of stationary armature-rotating field alternator?

- The current is drawn directly from fixed terminals on the stator without the use of brush contacts.
- The insulation of stationary armature winding is easier.
- The number of sliding contacts (slip rings) is reduced. Moreover, the sliding contacts are used for low-voltage DC Source.
- Armature windings can be constructed more rigidly to prevent deformation due to any mechanical stress.

18.What is single phase AC generator?

AC generator, which consists of single coil in armature produces single emf is known as single phase AC generator.

19.What is multi-phase AC generator?

AC generator, which consists of more than one coils in armature produces more than one emf is known as multi- phase AC generator.

20.What is two phase and three phase AC generator?

- AC generator, which consists of two coils in armature produces two emfs is known as two phase AC generator.
- AC generator, which consists of three coils in armature produces three emfs is known as three phase AC generator.

21.What are the advantages of three phase alternator?

- For a given dimension of the generator, three-phase machine produces higher power output than a single-phase machine.
- For the same capacity, three-phase alternator is smaller in size when compared to single phase alternator.
- Three-phase transmission system is cheaper. A relatively thinner wire is sufficient for transmission of three-phase power.

22.What is transformer? State its principle.

- Transformer is a stationary device used to transform electrical power from one circuit to another without changing its frequency.
- **Principle:** Mutual induction between two coils.

23.What are step-up and step-down transformers?

- If the transformer converts an alternating current with low voltage into an alternating current with high voltage, it is called step-up transformer.
- If the transformer converts alternating current with high voltage into an alternating current with low voltage, then it is called step-down transformer.

24.Define efficiency of a transformer.

The efficiency η of a transformer is defined as the ratio of the useful output power to the input power.

25.Write the energy losses in a transformer.

- Core loss or Iron loss.
- Copper loss.
- Flux leakage.

26.What is alternating voltage and alternating current?

- An alternating voltage is the voltage, which changes the polarity at regular intervals of time.

$$v = V_m sin\omega t$$

- An alternating current is the current, which changes its direction at regular intervals of time.

$$i = I_m sin\omega t$$

27.What is sinusoidal alternating voltage? Write its equation.

If the waveform of alternating voltage is a sine wave, then it is known as sinusoidal alternating voltage.

Equation: $v = V_m sin\omega t$

Here,

v – instantaneous value of alternating voltage.
V_m – maximum value of the voltage(amplitude).
ω - angular frequency of the alternating voltage.

28.Define average value of alternating current.

The average value of alternating current is defined as the average of all values of current over a positive half-cycle or negative half-cycle.

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m \ (for + ve\ half\ cycle)$$

$$I_{av} = -\frac{2I_m}{\pi} = -0.637 I_m \ (for - ve\ half\ cycle)$$

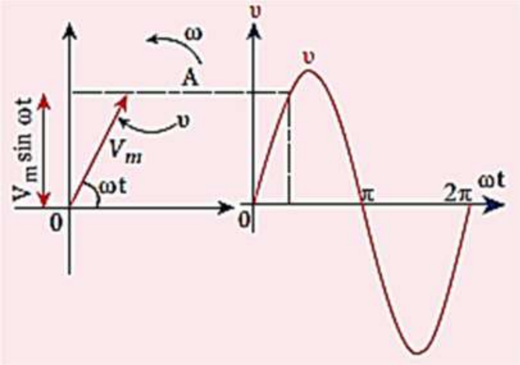
29.Define RMS value or effective value of alternating current(AC).

The root mean square value of an alternating current is defined as the square root of the mean of the squares of all currents over one cycle.

$$I_{RMS} = I_{eff} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

30.What is Phasor and Phasor diagram?

- A sinusoidal alternating voltage (or current) can be represented by a vector which rotates about the origin in anti-clockwise direction at a constant angular velocity ω. Such a rotating vector is called a **phasor.**



- The diagram which shows various phasors and their phase relations is called **phasor diagram.**

31.What is inductive reactance(X_L)? Give its unit.

The resistance offered by the inductor is called inductive reactance. Its unit is ohm.

$$X_L = \omega L = 2\pi f L$$

32.An inductor blocks AC but it allows DC. Why?

Since inductor is a closely wound helical coil, its self-induction produces induced emf during the AC current flow. This induced emf inturn opposes varying AC current according to Lenz's law. On the other hand DC current is not opposed as it doesn't have variation.

33.What is capacitive reactance(X_C)? Give its unit.

The resistance offered by the capacitor is called capacitive reactance. Its unit is ohm.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

34.An capacitor blocks DC but it allows AC. Why?

Not like for AC, the capacitor offers infinite resistance to steady current(DC). So that DC is blocked and AC is allowed in it.

35.What is electrical resonance and resonant frequency?

- When the frequency of the applied alternating source is equal to the natural frequency of the RLC circuit, the current becomes maximum. This condition is called **electrical resonance**.
- The frequency of the applied alternating source, at which resonance takes place is called **resonant frequency**.

$$\text{Resonant angular frequency}, \omega_r = \frac{1}{\sqrt{LC}}$$

$$\text{Resonant frequency}, f_r = \frac{1}{2\pi\sqrt{LC}}$$

36.What are the applications of series RLC circuit?

- Filter circuits
- Oscillators
- Voltage mutipliers
- Tuning circuits of radio and TV systems.

37.Define Q-factor or Quality factor.

Q-factor is defined as the ratio of voltage across L or C to the applied voltage.

38.What is power of a circuit?

Power of a circuit is defined as the rate of consumption of electric energy in that circuit. It depends on the components of the circuit.

39.What is wattful current and wattless current?

- The current component of AC $(I_{RMS}\cos\phi)$ has some power consumption value. So it is called wattful current.
- The current component of AC $(I_{RMS}\sin\phi)$ has zero power consumption value. So it is called wattless current.

40.Define Power factor.

- Power factor is defined as cosine of the angle of lead or lag(cosϕ).

(or)

- $\text{Power factor} = \frac{\text{Resistance}}{\text{Impedance}} = \frac{R}{Z}$

(or)

- $\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}} = \frac{VI\cos\phi}{VI}$

41.What are the advantages of AC over DC?

- The generation of AC is cheaper than that of DC.
- When AC is supplied at higher voltages, the transmission losses are small compared to DC transmission.
- AC can easily be converted into DC with the help of rectifiers.

42.What are the disadvantages of AC over DC?

- Alternating voltages cannot be used for certain applications e.g. charging of batteries, electroplating, electric traction etc.
- At high voltages, it is more dangerous to work with AC than DC.

43.What are LC oscillations?

Whenever energy is given to a LC circuit, the electrical oscillations of definite frequency are generated. These oscillations are called LC oscillations.

Conceptual Questions:

44.A spherical stone and a spherical metallic ball of same size and mass are dropped from the same height. Which one, a stone or a metal ball, will reach the earth's surface first? Justify your answer. Assume that there is no air friction?

- The stone will reach the earth's surface earlier than the metal ball.
- The reason is that when the metal ball falls through the magnetic field of earth, the eddy currents are produced in it which opposes its motion.
- But in the case of stone, no eddy currents are produced and it falls freely.

5. Electromagnetic Waves

1. What is displacement current?

The current which comes into play in the region in which the electric field and the electric flux are changing with time is called displacement current.

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

2. Write down the integral form of modified Ampere's circuital law.

Maxwell modified Ampere's law as,

$$\oint \vec{B} \cdot dl = \mu_0 I = \mu_0 (I_c + I_d)$$

Where,

I_c – Conduction current.
I_d – Displacement current.

3. What are electromagnetic waves?

A radiation comes from accelerated charge which propagates through space as coupled electric and magnetic fields oscillating perpendicular to each other and to the direction of propagation of wave is called electromagnetic wave.

4. What is poynting vector? Give its unit.

The rate of flow of energy crossing a unit area is known as poynting vector for electromagnetic waves. Its unit is W m^{-2}.

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = c^2 \varepsilon_0 (\vec{E} \times \vec{B})$$

5. Explain the concept of intensity of electromagnetic waves.

- The energy crossing per unit area per unit time and perpendicular to the direction of propagation of electromagnetic wave is called the intensity of electromagnetic wave.
- Intensity of electromagnetic wave is written as,

$$I = Average\ density \times velocity\ of\ light = \langle u \rangle\, c$$

(or)

$$I = \frac{\text{Total electromagnetic energy(U)}}{Surface\ area(A) \times Time(t)}$$

(or)

$$I = \frac{\text{Power(P)}}{Surface\ area(A)}$$

6. Write the classifications of electromagnetic spectrum.

- Radio waves
- Microwaves
- Infrared radiation
- Visible light
- Ultraviolet radiation
- X-rays
- Gamma rays

7. Write a short note on radio waves.

- It is produced by oscillators in electric circuits.
- The wavelength range is 1 X 10^{-1} m to 1 X 10^{4} m and frequency range is 3 X 10^{9} Hz to 3 X 10^{4} Hz.
- It obeys reflection and diffraction.

8. What are the uses of radio waves?

- It is used in radio and television communication systems.
- It is used in cellular phones to transmit voice communication in the ultra high frequency band.

9. Write a short note on microwaves.

- It is produced by electromagnetic oscillators in electric circuits.
- The wavelength range is 1 X 10^{-3} m to 3 X 10^{-1} m and frequency range is 3 X 10^{11} Hz to 1 X 10^{9} Hz.
- It obeys reflection and polarization.

10. What are the uses of microwaves?

- It is used in radar system for aircraft navigation, speed of the vehicle.
- It is used in microwave oven for cooking.
- It is used in very long distance wireless communication through satellites.

11. Write a short note on infrared radiation.

- It is produced from hot bodies (also known as heat waves) and also when the molecules undergo rotational and vibrational transitions.
- The wavelength range is 8 X 10^{-7} m to 5 X 10^{-3} m and frequency range is 4 X 10^{14} Hz to 6 X 10^{10} Hz.

12. What are the uses of infrared radiation?

- It provides electrical energy to satellites by means of solar cells.
- It is used to produce dehydrated fruits.
- It is used in green houses to keep the plants warm.
- It is used in heat therapy for muscular pain or sprain.
- It is used in TV remote as a signal carrier.
- It is used to look through haze fog or mist.
- It is used in night vision or infrared photography.

13. Write a short note on visible light.

- It is produced by incandescent bodies and also it is radiated by excited atoms in gases.
- The wavelength range is 4 X 10^{-7} m to 7 X 10^{-7} m and frequency range is 7 X 10^{14} Hz to 4 X 10^{14} Hz.
- It obeys the laws of reflection, refraction, interference, diffraction, polarization, photo-electric effect and photographic action.

14. What are the uses of visible light?

- It can be used to study the structure of molecules.
- It is used to arrangement of electrons in external shells of atoms.
- It is used to sensation of our eyes.

15. Write a short note on ultraviolet radiation.

- It is produced by Sun, arc and ionized gases.
- The wavelength range is 6 X 10^{-10} m to 4 X 10^{-7} m and frequency range is 5 X 10^{17} Hz to 7 X 10^{14} Hz.
- It has less penetrating power. It can be absorbed by atmospheric ozone and harmful to human body.

16. What are the uses of ultraviolet radiation?

- It is used to destroy bacteria and sterilizing the surgical instruments.
- It is used in burglar alarm.
- It is used to detect the invisible writing, finger prints.
- It is used in the study of molecular structure.

17. Write a short note on X-rays.

- It is produced when there is a sudden deceleration of high speed electrons at high-atomic number target, and also by electronic transitions among the innermost orbits of atoms.
- The wavelength range 10^{-13} m to 10^{-8} m and frequency range is 3 X 10^{21} Hz to 1 X 10^{16} Hz.
- X-rays have more penetrating power than ultraviolet radiation. X-rays are used extensively in studying structures of inner atomic electron shells and crystal structures.

18. What are the uses of X-rays?

- It is used in detecting fractures, diseased organs, formation of bones and stones, observing the progress of healing bones.
- It is used to detect faults, cracks, flaws and holes in a finished metal product.

19. Write a short note on gamma rays.

- It is produced by transitions of atomic nuclei and decay of certain elementary particles.
- The wavelength range is 1 X 10^{-14} m to 1 X 10^{-10} m and frequency range is 3 X 10^{22} Hz to 3 X 10^{18} Hz.
- They produce chemical reactions on photographic plates, fluorescence, ionisation, diffraction.
- Gamma rays have high penetrating power than X-rays and ultraviolet radiations; it has no charge but harmful to human body.

20. What are the uses of gamma rays?

- It is used to provide information about the structure of atomic nuclei.
- It is used in radiotherapy for the treatment of cancer and tumour.
- It is used in food industry to kill pathogenic micro-organism.

21. What is emission spectra? Name its types.

- The spectrum of self luminous source is called emission spectrum.
- **Types:**
    - Continuous emission spectra
    - Line emission spectra.
    - Band emission spectra.

22. What is absorption spectra? Name its types.

- When light is allowed to pass through a medium or an absorbing substance then the spectrum obtained is known as absorption spectrum.
- **Types:**
    - Continuous absorption spectra
    - Line absorption spectra.
    - Band absorption spectra.

23. What is meant by Fraunhofer lines?

The dark lines observed in the solar line absorption spectrum are known as Fraunhofer lines.

5 Marks Q & A:

1. Write down about Maxwell equations in integral form.

- Electrodynamics can be summarized into four basic equations, known as Maxwell's equations.
- These equations are analogous to Newton's equations in mechanics.
- Maxwell's equations completely explain the behaviour of charges, currents and properties of electric and magnetic fields.

(a) Maxwell's 1st equation:(Gauss's law in electrostatics)

- First equation is nothing but the Gauss's law. It relates the net electric flux to net electric charge enclosed in a surface. Mathematically, it is expressed as,

$$\oint \vec{E} \,.\, d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

where $\vec{E}$ is the electric field and $Q_{enclosed}$ is the charge enclosed.

- This equation is true for both discrete and continuous distribution of charges. It also indicates that the electric field lines start from positive charge and terminate at negative charge.
- This implies that the electric field lines do not form a continuous closed path. In other words, it means that isolated positive charge or negative charge can exist.

(b) Maxwell's 2nd equation:(Gauss's law in magnetism)

- The surface integral of magnetic field over a closed surface is zero. Mathematically,

$$\oint \vec{B} \,.\, d\vec{A} = 0$$

where $\vec{B}$ is the magnetic field.

- This equation implies that the magnetic lines of force form a continuous closed path. In other words, it means that no isolated magnetic monopole exists.

(c) Maxwell's 3rd equation:
(Faraday's law of electromagnetic induction)

- This law relates electric field with the changing magnetic flux which is mathematically written as,

$$\oint \vec{E} \,.\, d\vec{l} = \frac{d\Phi_B}{dt}$$

where $\vec{E}$ is the electric field.

- This equation implies that the line integral of the electric field around any closed path is equal to the rate of change of magnetic flux through the closed path bounded by the surface.

(d) Maxwell's 4th equation:(Ampere-Maxwell's law)

- This law relates the magnetic field around any closed path to the conduction current and displacement current through that path.

$$\oint \vec{B} \,.\, d\vec{l} = \mu_0 I_{enclosed} + \mu_0 \varepsilon_0 \oint_s \vec{E} \cdot d\vec{A}$$

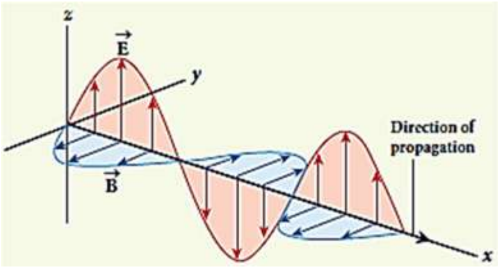
$$\oint \vec{B} \,.\, d\vec{l} = \mu_0 I_{enclosed} + \mu_0 I_d$$

where $\vec{B}$ is the magnetic field.

- This equation shows that both conduction and also displacement current produces magnetic field.
- These four equations are known as Maxwell's equations in electrodynamics. This equation ensures the existence of electromagnetic waves.

2. Write down the properties of electromagnetic waves.

- Electromagnetic waves are produced by any accelerated charge.
- Electromagnetic waves do not require any medium for propagation. So electromagnetic wave is a non-mechanical wave.
- Electromagnetic waves are transverse in nature. This means that the oscillating electric field vector, oscillating magnetic field vector and propagation vector (gives direction of propagation) are mutually perpendicular to each other.



- Electromagnetic waves travel with speed which is equal to the speed of light in vacuum or free space,

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \times 10^8 ms^{-1}$$

Where ε_o is the permittivity of free space or vacuum and μ_0 is the permeability of free space or vacuum

- The speed of electromagnetic wave in a medium (with permittivity ε and permeability μ) is less than speed in free space or vacuum, i.e. v < c.

Therefore, in a medium of refractive index,

$$\mu = \frac{c}{v} = \frac{\frac{1}{\sqrt{\varepsilon_0 \mu_0}}}{\frac{1}{\sqrt{\varepsilon \mu}}} = \sqrt{\varepsilon_r \mu_r}$$

Where ε_r and μ_r are the relative permittivity and permeability of the medium repectively.

- Electromagnetic waves are not deflected by electric field or magnetic field.

- Electromagnetic waves can show interference, diffraction and can also be polarized.
- The energy density (energy per unit volume) associated with an electromagnetic wave propagating in vacuum or free space is,

$$u = \left\{\begin{matrix} energy\ density \\ in\ electric\ field(u_E) \end{matrix}\right\} + \left\{\begin{matrix} energy\ density \\ in\ magnetic\ field(u_B) \end{matrix}\right\}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$

Since $E = B\,c,\ u_E = u_B$,There fore,

$$u = \varepsilon_0 E^2 = \frac{1}{\mu_0}B^2$$

- The average energy density for electromagnetic wave,

$$\langle u\rangle = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2\mu_0}B^2$$

- The energy crossing per unit area per unit time and perpendicular to the direction of propagation of electromagnetic wave is called the intensity of electromagnetic wave.
- Intensity of electromagnetic wave is written as,

$$I = Average\ density \times velocity\ of\ light = \langle u\rangle\, c$$

(or)

$$I = \frac{\text{Total electromagnetic energy(U)}}{Surface\ area(A) \times Time(t)}$$

(or)

$$I = \frac{\text{Power(P)}}{Surface\ area(A)}$$

- Like other waves, electromagnetic waves also carry energy and momentum.

$$\left\{\begin{matrix} Linear\ momentum\ of \\ electromanetic\ wave \end{matrix}\right\} p = \frac{Energy}{Speed} = \frac{U}{c}$$

- If the electromagnetic wave incident on a material surface is completely absorbed, then the energy delivered is U and momentum imparted on the surface is $p = \frac{U}{c}$.
- If the incident electromagnetic wave of energy U is totally reflected from the surface, then the momentum delivered to the surface is,

$$\Delta p = \frac{U}{c} - \left(-\frac{U}{c}\right) = 2\frac{U}{c}$$

- The rate of flow of energy crossing a unit area is known as poynting vector for electromagnetic waves, which is

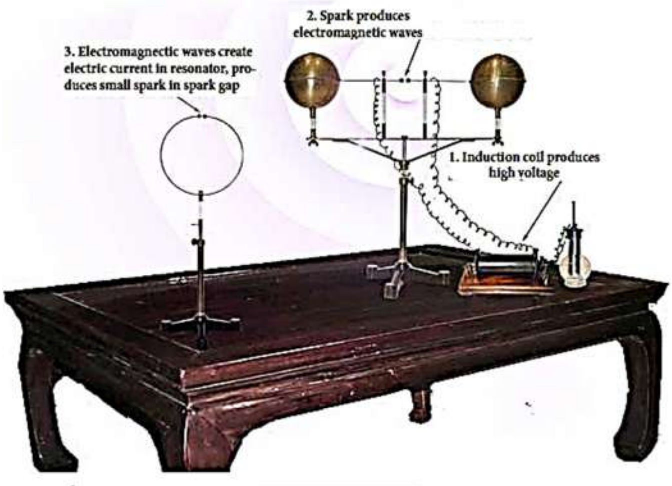
$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = c^2\varepsilon_0(\vec{E} \times \vec{B})$$

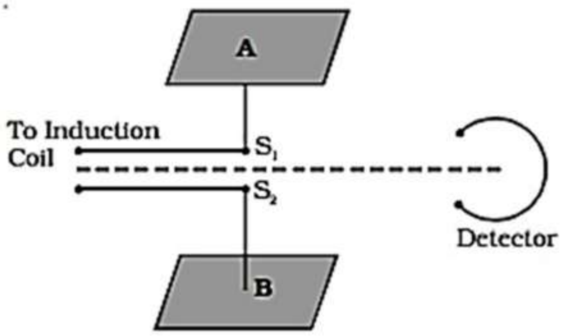
The unit for poynting vector is W m^{-2}. The poynting vector at any point gives the direction of energy transport from that point.

- Electromagnetic waves carries not only energy and momentum but also angular momentum.

3✓ Discuss briefly the experiment conducted by Hertz to produce and detect electromagnetic spectrum.

- It consists of two metal electrodes which are made of small spherical metals as shown in figure.

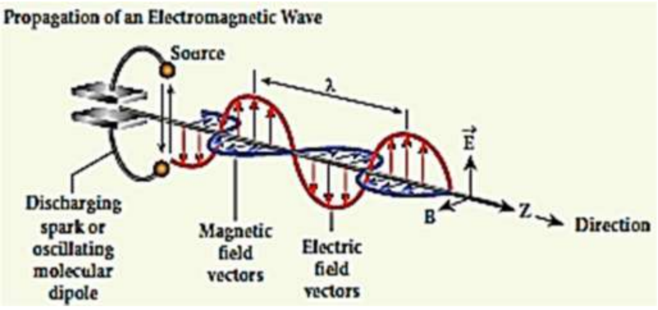




- These are connected to larger spheres and the ends of them are connected to induction coil with very large number of turns. This is to produce very high electromotive force (emf).
- Since the coil is maintained at very high potential, air between the electrodes gets ionized and spark (spark means discharge of electricity) is produced.
- The gap between electrode (ring type – not completely closed and has a small gap in between) kept at a distance also gets spark.
- This implies that the energy is transmitted from electrode to the receiver (ring electrode) as a wave, known as electromagnetic waves.
- If the receiver is rotated by 90^0 - then no spark is observed by the receiver.
- This confirms that electromagnetic waves are transverse waves as predicted by Maxwell.
- Hertz detected radio waves and also computed the speed of radio waves which is equal to the speed of light (3 X 10^8 m s^{-1}).

4. Discuss the source of electromagnetic waves.

- ❖ Any stationary source charge produces only electric field .
- ❖ When the charge moves with uniform velocity, it produces steady current which gives magnetic field (not time dependent, only space dependent) around the conductor in which charge flows.
- ❖ If the charged particle accelerates, in addition to electric field it also produces magnetic field. Both electric and magnetic fields are time varying fields.
- ❖ An oscillatory or accelerating motion of charge (oscillating molecular dipole) about their mean position, produces electromagnetic waves as shown in figure.



- ❖ Since the electromagnetic waves are transverse waves, the direction of propagation of electro magnetic waves is perpendicular to the plane containing electric and magnetic field vectors.
- ❖ Suppose the electromagnetic field in free space propagates along z direction, the electric field vector points along y axis and the magnetic field vector points along x direction, then,

$$E_y = E_0 \sin(kz - \omega t)$$
$$B_x = B_0 \sin(kz - \omega t)$$

where,

E_o & B_o - Amplitude of oscillating electric and magnetic field.
k - wave number.
ω - The angular frequency of the wave.
$\hat{k}$ - unit vector, here it is called propagation vector which denotes the direction of propagation of electromagnetic wave.

- ❖ Here, frequency of electro magnetic wave is equal to the frequency of the source (oscillating charge).
- ❖ In free space or in vacuum, the ratio between E_o and B_o is equal to the speed of electromagnetic wave, which is equal to speed of light c.

$$c = \frac{E_0}{B_0}$$

- ❖ In any medium, the ratio of E_o and B_o is equal to the speed of electromagnetic wave in that medium, mathematically, it can be written as,

$$v = \frac{E_0}{B_0} < c$$

- ❖ Further, the energy of electromagnetic waves comes from the energy of the oscillating charge.

5. What is emission spectra?. Explain their types.

- ❖ When the spectrum of self luminous source is taken, we get emission spectrum. Each source has its own characteristic emission spectrum.
- ❖ **Types of emission spectrum:**
  - Continuous emission spectra (or continuous spectra)
  - Line emission spectrum (or line spectrum)
  - Band emission spectrum (or band spectrum)

(i) Continuous emission spectra (or continuous spectra):

- ❖ If the light from incandescent lamp (filament bulb) is allowed to pass through prism (simplest spectroscope), it splits into seven colours.
- ❖ Thus, it consists of wavelengths containing all the visible colours ranging from violet to red as shown in figure.

- ❖ <u>Examples:</u> spectrum obtained from carbon arc, incandescent solids, liquids gives continuous spectra.

(ii) Line emission spectrum (or line spectrum):

- ❖ Suppose light from hot gas is allowed to pass through prism, line spectrum is observed as shown in figure.

- ❖ Line spectra are also known as discontinuous spectra.
- ❖ The line spectra are sharp lines of definite wavelengths or frequencies. Such spectra arise due to excited atoms of elements.
- ❖ These lines are the characteristics of the element which means it is different for different elements.
- ❖ <u>Examples:</u> spectra of atomic hydrogen, helium, etc.

(iii) Band emission spectrum (or band spectrum):

- ❖ Band spectrum consists of several number of very closely spaced spectral lines which overlapped together forming specific bands which are separated by dark spaces, known as band spectra.

- ❖ This spectrum has a sharp edge at one end and fades out at the other end. Such spectra arise when the molecules are excited.
- ❖ Band spectrum is the characteristic of the molecule hence, the structure of the molecules can be studied using their band spectra.
- ❖ <u>Examples:</u> spectra of hydrogen gas, ammonia gas in the discharge tube etc.

6) What is absorption spectra? Explain their types.

- When light is allowed to pass through a medium or an absorbing substance then the spectrum obtained is known as absorption spectrum.

- It is the characteristic of absorbing substance.

- **Types of absorption spectrum:**
  - Continuous absorption spectrum.
  - Line absorption spectrum.
  - Band absorption spectrum.

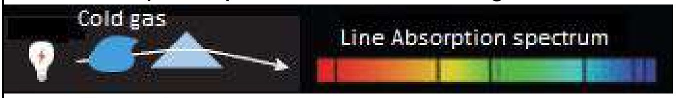
(i) Continuous absorption spectrum:

- When the light passed through a medium is dispersed by the prism, we get continuous absorption spectrum.

- __Example:__ when we pass white light through a blue glass plate, it absorbs everything except blue.

(ii) Line absorption spectrum:

- When light from the incandescent lamp is passed through cold gas (medium), the spectrum obtained through the dispersion due to prism is line absorption spectrum as shown in figure.



- __Example:__If the light from the carbon arc is made to pass through sodium vapour, a continuous spectrum of carbon arc with two dark lines in the yellow region of sodium vapour is obtained.

(iii) Band absorption spectrum:

- When the white light is passed through the iodine vapour, dark bands on continuous bright background is obtained.

- __Example:__ when white light is passed through diluted solution of blood or chlorophyll or through certain solutions of organic and inorganic compounds, band absortion spectrum is obtained.